# Rational function word problems with solutions pdf

In Example 2, we shifted a toolkit function in a way that resulted in the function [latex]f\left(x\right)=\frac{3x+7}{x+2}[/latex]. This is an example of a rational function. A rational function is a function that can be written as the quotient of two polynomial functions. Many real-world problems require us to find the ratio of two polynomial functions. Problems involving rates and concentrations often involve rational functions. A rational function is a function that can be written as the quotient of two polynomial functions [latex]P\left(x\right) \text{and} Q\left(x\right)[/latex]. [latex]f\left(x\right)=\frac{P\left(x\right)}{Q\left(x\right)}=\frac{{a}_{p}{x}^{p}+{a}_{p - 1}{x}^{p - 1}+...+{a}_{1}x+{a}_{0}}{{b}_{q}{x}^{q}+{b}_{q - 1}{x}^{q - 1}+...+{b}_{1}x+{b}_{0}},Q\left(x\right)e 0[/latex] A large mixing tank currently contains 100 gallons of water into which 5 pounds of sugar have been mixed. A tap will open pouring 10 gallons per minute of water into the tank at the same time sugar is poured into the tank at a rate of 1 pound per minute. Find the concentration (pounds per gallon) of sugar in the tank after 12 minutes. Is that a greater concentration than at the beginning? Let t be the number of minutes since the tap opened. Since the water increases at 10 gallons per minute, and the sugar increases at 1 pound per minute, these are constant rates of change. This tells us the amount of water in the tank is changing linearly, as is the amount of sugar in the tank. We can write an equation independently for each: [latex]\begin{cases}\text{water: }W\left(t\right)=100+10t\text{ in gallons}\\ \text{sugar: }S\left(t\right)=5+1t\text{ in pounds}\end{cases}[/latex] The concentration, C, will be the ratio of pounds of sugar to gallons of water [latex]C\left(t\right)=\frac{5+t}{100+10t}[/latex] The concentration after 12 minutes is given by evaluating [latex]C\left(t\right)[/latex] at [latex]t=\text{ }12[/latex]. [latex]\begin{cases}C\left(12\right)=\frac{5+12}{100+10\left(12\right)}\hfill \\ \text{ }=\frac{17}{220}\hfill \end{cases}[/latex] This means the concentration is 17 pounds of sugar to 220 gallons of water. At the beginning, the concentration is [latex]\begin{cases}C\left(0\right)=\frac{5+0}{100+10\left(0\right)}\hfill \\ \text{ }=\frac{1}{20}\hfill \end{cases}[/latex] Since [latex]\frac{17}{220}\approx 0.08>\frac{1}{20}=0.05[/latex], the concentration is greater after 12 minutes than at the beginning. There are 1,200 freshmen and 1,500 sophomores at a prep rally at noon. After 12 p.m., 20 freshmen arrive at the rally every five minutes while 15 sophomores leave the rally. Find the ratio of freshmen to sophomores at 1 p.m. Solution A nice application of rational functions involves the amount of work a person (or team of persons) can do in a certain amount of time. We can handle these applications involving work in a manner similar to the method we used to solve distance, speed, and time problems. Here is the guiding principle. Note The amount of work done is equal to the product of the rate at which work is being done and the amount of time required to do the work. That is, \[\text { Work }=\text { Rate } \times \text { Time. }\] For example, suppose that Emilia can mow lawns at a rate of 3 lawns per hour. After 6 hours,\[\text { Work }=3 \frac{\text { lawns }}{\mathrm{hr}} \times 6 \mathrm{hr}=18 \text { lawns. }\]A second important concept is the fact that rates add. For example, if Emilia can mow lawns at a rate of 3 lawns per hour and Michele can mow the same lawns at arate of 2 lawns per hour, then together they can mow the lawns at a combined rate of 5 lawns per hour.Let's look at an example. Example \(\PageIndex{5}\) Bill can finish a report in 2 hours. Maria can finish the same report in 4 hours. How long will it take them to finish the report if they work together? Solution A common misconception is that the times add in this case. That is, it takes Bill 2 hours to complete the report and it takes Maria 4 hours to complete the same report, so if Bill and Maria work together it will take 6 hours to complete the report. A little thought reveals that this result is nonsense. Clearly, if they work together, it will take them less time than it takes Bill to complete the report alone; that is, the combined time will surely be less than 2 hours. However, as we saw above, the rates at which they are working will add. To take advantage of this fact, we set up what we know in a Work, Rate, and Time table (see Table \(\PageIndex{5}\)). • It takes Bill 2 hours to complete 1 report. This is reflected in the entries in the first row of Table \(\PageIndex{5}\). • It takes Maria 4 hours to complete 1 report. This is reflected in the entries in the second row of Table \(\PageIndex{5}\). • Let t represent the time it takes them to complete 1 report if they work together. This is reflected in the entries in the last row of Table \(\PageIndex{5}\). w (reports) r (reports/h) t(h) Bill 1 ? 2 Maria 1 ? 4 Together 1 ? t Table \(\PageIndex{5}\). A work, rate, and time table. We have advice similar to that given for distance, speed, and time tables. Work, Rate, and Time Tables Because work, rate, and time are related by the equation \[\text { Work }=\text { Rate } \times \text { Time }\] whenever you have two boxes in a row completed, the third box in that row can be calculated by means of the relation Work \(=\) Rate \(\times\) Time. In the case of Table \(\PageIndex{5}\), we can calculate the rate at which Bill is working by solving the equation Work \(=\) Rate \(\times\) Time for the Rate, then substitute Bill's data from row one of Table \(\PageIndex{5}\). \[Rate \(=\frac{\text { Work }}{\text { Time }}=\frac{1 \text { report }}{2 \mathrm{h}}\)\] Thus, Bill is working at a rate of 1/2 report per hour. Note how we've entered this result in the first row of Table 6. Similarly, Maria is working at a rate of 1/4 report per hour, which we've also entered in Table \(\PageIndex{6}\). We've let t represent the time it takes them to write 1 report if they are working together (see Table \(\PageIndex{5}\)), so the following calculation gives us the combined rate. \[Rate \(=\frac{\text { Work }}{\text { Time }}=\frac{1 \text { report }}{t \mathrm{h}}\)\] That is, together they work at a rate of 1/t reports per hour. This result is also recorded in Table \(\PageIndex{6}\). w (reports) r (reports/h) t(h) Bill 1 1/2 2 Maria 1 1/4 4 Together 1 1/t t Table \(\PageIndex{6}\). Calculating the Rate entries. In our discussion above, we pointed out the fact that rates add. Thus, the equation we seek lies in the Rate column of Table \(\PageIndex{6}\). Bill is working at a rate of 1/2 report per hour and Maria is working at a rate of 1/4 report per hour. Therefore, their combined rate is 1/2 + 1/4 reports per hour. However, the last row of Table \(\PageIndex{6}\) indicates that the combined rate is also 1/t reports per hour. Thus, \[\frac{1}{2}+\frac{1}{4}=\frac{1}{t}\] Multiply both sides of this equation by the common denominator 4t. \[\begin{aligned}\color{blue}{(4 t)}\left[\frac{1}{2}+\frac{1}{4}\right] &=\left[\frac{1}{t}\right]\color{blue}{(4 t)} \\ 2 t+t &=4 \end{aligned}\] This equation is linear (no power of t other than 1) and is easily solved. \[\begin{aligned} 3 t &=4 \\ t &=4 / 3 \end{aligned}\] Thus, it will take 4/3 of an hour to complete 1 report if Bill and Maria work together. Again, it is very important that we check this result. • We know that Bill does 1/2 reports per hour. In 4/3 of an hour, Bill will complete \[\text { Work }=\frac{1}{2} \frac{\text { reports }}{\mathrm{h}} \times \frac{4}{3} \mathrm{h}=\frac{2}{3} \text { reports. }\] That is, Bill will complete 2/3 of a report. • We know that Maria does 1/4 reports per hour. In 4/3 of an hour, Maria will complete \[\text { Work }=\frac{1}{4} \frac{\text { reports }}{\mathrm{h}} \times \frac{4}{3} \mathrm{h}=\frac{1}{3} \mathrm{reports}\] That is, Maria will complete 1/3 of a report. Clearly, working together, Bill and Maria will complete 2/3 + 1/3 reports, that is, one full report. Let's look at another example. Example \(\PageIndex{6}\) It takes Liya 7 more hours to paint a kitchen than it takes Hank to complete the same job. Together, they can complete the same job in 12 hours. How long does it take Hank to complete the job if he works alone? Solution Let H represent the time it take Hank to complete the job of painting the kitchen when he works alone. Because it takes Liya 7 more hours than it takes Hank, let H + 7 represent the time it takes Liya to paint the kitchen when she works alone. This leads to the entries in Table \(\PageIndex{7}\). w (kitchens) r (kitchens/h) t(h) Hank 1 ? H Liya 1 ? H+7 Together 1 ? 12 Table \(\PageIndex{7}\). Entering the given data for Hank and Liya. We can calculate the rate at which Hank is working alone by solving the equation Work \(=\) Rate \(\times\) Time for the Rate, then substituting Hank's data from row one of Table \(\PageIndex{7}\). \[\text { Rate }=\frac{\text { Work }}{\text { Time }}=\frac{1 \text { kitchen }}{H \text { hour }}\] Thus, Hank is working at a rate of 1/H kitchens per hour. Similarly, Liya is working at a rate of 1/(H + 7) kitchens per hour. Because it takes them 12 hours to complete the task when working together, their combined rate is 1/12 kitchens per hour. Each of these rates is entered in Table \(\PageIndex{8}\). w (kitchens) r (kitchens/h) t(h) Hank 1 1/H H Liya 1 1/H+7 H+7 Together 1 1/12 12 Table \(\PageIndex{8}\). Calculating the rates. Because the rates add, we can write \[\frac{1}{H}+\frac{1}{H+7}=\frac{1}{12}\] Multiply both sides of this equation by the common denominator 12H(H + 7). \[\begin{aligned} \color{blue}{12 H(H+7)}\left(\frac{1}{H}+\frac{1}{H+7}\right) &=\left(\frac{1}{12}\right)\color{blue}{12 H(H+7)} \\ 12(H+7)+12 H &=H(H+7) \end{aligned}\] Expand and simplify. \[\begin{aligned} 12 H+84+12 H &=H^{2}+7 H \\ 24 H+84 &=H^{2}+7 H \end{aligned}\] This last equation is nonlinear, so make one side zero by subtracting 24H and 84 from both sides of the equation. \[\begin{array}{l}{0=H^{2}+7 H-24 H-84} \\ {0=H^{2}-17 H-84}\end{array}\] Note that ac = (1)(−84) = −84. The integer pair {4, −21} has product −84 and sums to −17. Hence, \[0=(H+4)(H-21)\] Using the zero product property, either \[H+4=0 \quad \text { or } \quad H-21=0\] leading to the solutions \[H=-4 \quad \text { or } \quad H=21\] We eliminate the solution H = −4 from consideration (it doesn't take Hank negative time to paint the kitchen), so we conclude that it takes Hank 21 hours to paint the kitchen. Does our solution make sense? It takes Hank 21 hours to complete the kitchen, so he is finishing 1/21 of the kitchen per hour. It takes Liya 7 hours longer than Hank to complete the kitchen, namely 28 hours, so she is finishing 1/28 of the kitchen per hour. Together, they are working at a combined rate of \[\frac{1}{21}+\frac{1}{28}=\frac{4}{84}+\frac{3}{84}=\frac{7}{84}=\frac{1}{12}\] or 1/12 of a kitchen per hour. This agrees with the combined rate in Table \(\PageIndex{8}\).

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