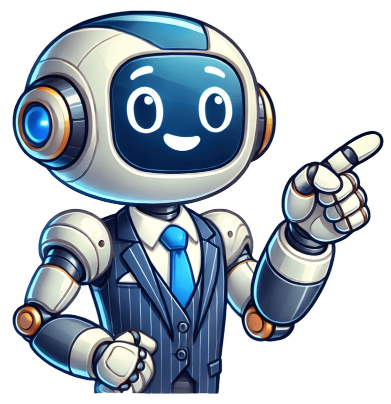

A 30°-60-90 triangle is a special right triangle in which the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is the square root of 3 times the length of the shorter leg.Hypotenuse = 2 × shorter legLonger leg = √3 × shorter leg Looking carefully at the figure above, you may have observed the following ratios:Suppose we start from the smallest angle to the largest angle and from the shortest side to the longest sideThe angles of a 30-60-90 triangle are in the ratio 1:2:3The sides of a 30-60-90 triangle are in the ratio 1:√3:2 Start with an equilateral triangle ABC where each interior angle is equal to 60 degrees.Since segment BD is the perpendicular bisector of segment AC, we now have two 30-60-90 triangles.Let segment AD equal to s and use the Pythagorean theorem to find the length of segment BD.Notice that AC = s + s = 2s. Since AB = AC, AB = 2s.AD2 + BD2 = AB2s2 + BD2 = (2s)2s2 - s2 + BD2 = (2s)2 - s2BD2 = 4s2 - s2BD2 = 3s2BD = √(3s2)BD = s√3 Examples of 30-60-90 triangles Example #1: 1, √3, 2Short leg: 1long leg: √3Hypotenuse: 2Example #2: 2, 2√3, 4Short leg: 2long leg: 2√3Hypotenuse: 4Example #3: 5, 5√3, 10Short leg: 5long leg: 5√3Hypotenuse: 10 Using the length of one side to solve a 30-60-90 triangle Example #4:The longer leg of a 30-60-90 triangle is 5. Find the lengths of the other sides.Longer leg = √3 × shorter leg5 = √3 × shorter legShorter leg = 5/√3 Multiply both numerator and denominator of 5/√3 by √3Shorter leg = 5(√3) / √3(√3) Shorter leg = 5(√3) / √9Shorter leg = 5(√3) / 3Hypotenuse = 2 × shorter legHypotenuse = 2 × [5(√3) / 3]Hypotenuse = 10√3 / 3 Using the 30-60-90 triangle to solve real-world problems Example #5:The warning sign below is an equilateral triangle. Suppose the height of the sign is 1 meter. Find the length s of each side of the sign. Since the triangle is an equilateral triangle, the altitude divides the triangle into two 30-60-90 triangles as shown in the figure below. Notice that the length of the shorter leg is equal to (1/2)s or 0.5s since the altitude also bisects the base.1 = √3 × shorter leg1 = √3 × 0.5sDivide both sides of the equation by 0.51/0.5 = (√3)s2 = (√3)sDivide both sides of the equation by (√3)2/(√3) = ss = 1.15 mThe length of each side of the sign is about 1.2 m Example #6:An escalator lifts people to the second floor, 40 feet above the first floor. The escalator rises at a 30 degrees angle. How far does the person travel from the bottom to the top of the escalator? The figure above describes example #6 with a 30-60-90 triangle. Notice that the shorter side is 40 feet and it is opposite to the angle that measures 30 degrees.The distance from the bottom of the escalator to the top is the length of the hypotenuse of the 30-60-90 triangle. This distance is twice the length of the shorter side. Therefore, the distance you travel is 80 feet. Area of a 30-60-90 triangle To find the area of a 30-60-90 triangle, just make sure you get the height of the triangle correctly.Keep in mind that either the longer side or the shorter side could be the height of the triangle. Just use the formula below:Longer leg = √3 × shorter legIf the shorter leg of the triangle is given, then the height = √3 × shorter leg and the shorter leg is the base.If the longer leg of the triangle is given, then the height = longer leg / √3 and the base is the longer leg. Example #7:Find the area of the 30-60-90 triangle below Since the longer leg is the base, height = 8/√3 = 4.618Area = 8 × 4.618Area = 36.944The area of the triangle is 36.944 cm2 Hanna Pamuła, PhDHanna (Hania) Pamuła holds a Ph.D. in Bioacoustics / Mechanical Engineering, obtained at AGH University of Science and Technology. She has participated in research work in labs in France and the UK and presented papers at several international conferences. Hania has a penchant for photography and graphic design. When not in the office, she’s probably traveling, hiking, or out in the field, watching birds and recording their calls. See full profileCheck our editorial policyBogna Szyk and Adena BennAdena Benn is a Guyanese teacher with a degree in computer science who is always reading and learning. She loves problem-solving, everything tech, and working with teenagers. She has a passion for education and is especially interested in how children learn and the teaching methods that best suit their learning styles. She grew up on a farm in Pomeroon, Guyana, where she worked alongside her parents and siblings. As such, she is just as comfortable growing plants as teaching in the classroom. In her early life, she also gained expertise as a seamstress, which she learned from her mother. By grade 9, she had already acquired her dressmaker's certificate. Today she uses her skills to design many items for her family. In her free time, Adena loves to read, take long walks, write children’s stories and poetry, travel, or spend time with her family. See full profileCheck our editorial policy1 504 people find this calculator helpfulCheck out 19 similar triangle calculators A three sided polygon is known as a triangle. A triangle has 3 sides, 3 vertices and 3 angles. There are different ways in which we can categorize the triangles. On the basis of sides, a triangle can be scalene, isosceles, or equilateral. However, in some cases, a triangle may possess unique properties. Such triangles are called special triangles. One such special triangle is the 30°–60°–90° triangles. A special right triangle with angles 30°, 60°, and 90° is called a 30-60-90 triangle. The angles of a 30-60-90 triangle are in the ratio 1 : 2 : 3. Since 30° is the smallest angle in the triangle, the side opposite to the 30° angle is always the smallest (shortest leg). The side opposite to the 60° angle is the longer leg, and finally, the side opposite to the 90° angle is the largest side of the right triangle, also known as the hypotenuse. Since the 30-60-90 triangle is a special triangle, the side lengths of the 30-60-90 triangle are in a constant relationship. From the figure above, we can make the following observations about the side length ratio of a 30-60-90 triangle: The side opposite the 30° angle: PQ = a The side opposite the 60° angle: QR = a√3 The side opposite the 90° angle: PR = 2a The sides of a 30-60-90 triangle are always in the ratio of 1 : √3 : 2. For example: Here, in triangle PQR, The side opposite to the 30° angle is PQ = a = 5 units The side opposite to the 60° angle is QR = a√3 = 5√3 units The side opposite to the 90° angle is PQ = 2a = 10 units In a 30-60-90 triangle, we can find the measure of any of the three sides by knowing the measure of at least one side in the triangle. This is known as the 30-60-90 triangle rule. The following shows how to find the sides of a 30-60-90 triangle using this rule: When the side opposite to 30° is given. Consider the side opposite to 30° (shortest side), AB to be ‘a’. The side opposite to 60°, BC = a√3 The side opposite to 90° (hypotenuse), AC = 2a When the side opposite to 60° is given. Consider the side opposite to 60°, EF to be ‘a’. The side opposite to 30°, DE = $\frac{a}{\sqrt{3}}$ The side opposite to 90° (hypotenuse), DF = $\frac{2a}{\sqrt{3}}$ When hypotenuse is given. Consider the hypotenuse of the triangle, i.e., PR to be ‘a’. The side opposite to 30°, PQ = $\frac{a}{2}$ The side opposite to 60°, QR = $\frac{\sqrt{3}}{2}$a More Worksheets When the perpendicular is given to be a, Area of ΔDEF = $\frac{1}{2} \times base \times height$ = $\frac{1}{2} \times a \sqrt{3} \times a = \frac{\sqrt{3}a^{2}}{2}$ In this article, we learned about a special triangle, i.e., the 30-60-90 triangle. To read more such informative articles on other concepts, visit our website. We, at SplashLearn, are on a mission to make learning fun and interactive for all students. 1. Find the length of the side BC in the following figure. Answer: The side opposite to 30˚ (shortest side), AB = 6 cm Since the sides of a 30-60-90 triangle are always in the ratio of 1 : √3 : 2. Therefore, the sides of the given triangle are AB = 6 cm, BC = 6√3 cm and AC = 12 cm. So, BC = 6√3 cm 2. Find the length of the hypotenuse in the following figure. Answer: Side opposite to 60˚, QR = a = 8√3 cm The hypotenuse of the triangle, i.e., PR = $\frac{2a}{\sqrt{3}} = \frac{2 × 8√3}{\sqrt{3}}$ = 16 units. 3. A triangle has sides 3√2, 3√6, and 3√8. Find the angles of this triangle. Answer: The sides of the triangle are 3√2, 3√6, and 3√8. Let us check whether the sides are of the 30-60-90 triangle. On dividing each side of the triangle by 3√2, we get 1, √3, and 2. Therefore, sides of the triangle are 3√2, 3√6, and 3√8, are in the ratio 1: √3: 2. The sides are following the 30-60-90 triangle rule. So, the angles of the given triangle are 30°, 60°, and 90°. Attend this Quiz & Test your knowledge.Correct answer is: 5, 5√3, 10On dividing the ratio 5 : 5√3 : 10 by 5, we get 1 : √3 : 2. It follows the 30-60-90 triangle rule.$\frac{10 \sqrt{3}}{3}$ feetCorrect answer is: 10 feetIn a 30-60-90 triangle, the length of the hypotenuse is twice the length of the shortest side. So, the shortest side of the triangle must be $\frac{hypotenuse}{2}$ = $\frac{20}{2}$ = 10 ftCorrect answer is: 8$\sqrt{3}$ cm^{2}Base is 4√3 cm and perpendicular must be 4 cm. So, Area of the triangle = $\frac{1}{2} \times 4 \times 4√3 = 8√3 cm^{2}$ How is the 30-60-90 triangle similar to 45-45-90? These are some similarities between the 30-60-90 triangle and 45-45-90 triangle. Both are right-angle triangles. Both follow Pythagorean theorem. Sum of the interior angles of both are 180 degrees. What is the perimeter of a 30-60-90 triangle? Let the perpendicular be a. Base = 3a Hypotenuse = 2a Perimeter = Sum of all the sides = a+3a+2a = a(3+3) units Which side is the longer leg of a 30-60-90 triangle? The long leg of a 30-60-90 triangle is the one whose length is greater than the shortest leg and less than the hypotenuse. In other words, it is the side opposite the 60 degree angle. A 30-60-90 triangle is a special right-angled triangle as the angles of the triangle are in the ratio 1:2:3. There are different types of triangles such as obtuse, isosceles, acute, equilateral, and so on. But only a few types of triangles are considered special triangles. These triangles are special as their sides and angles are consistent and predictable. Their properties can be used to solve various geometry or trigonometry problems. A 30-60-90 triangle—pronounced "thirty sixty ninety"—is one such very special type of triangle indeed. In this lesson, we will explore the concept of the 30-60-90 triangle and learn all about it including its formula, definition, sides, area, and the rules that apply to this triangle. What is 30-60-90 Triangle? The 30-60-90 triangle is called a special right triangle as the angles of this triangle are in a unique ratio of 1:2:3. Here, a right triangle means being any triangle that contains a 90° angle. A 30-60-90 triangle is a special right triangle that always has angles of measure 30°, 60°, and 90°. Here are some of the variants of a 30-60-90 triangle. The triangles ABC and PQK are 30-60-90 triangles. Here, in the triangle ABC, ∠ C = 30°,∠ A = 60°, and ∠ B = 90° and in the triangle PQK, ∠ P = 30°,∠ K = 60°, and ∠ Q = 90° 30-60-90 Triangle Sides A 30-60-90 triangle is a special triangle since the length of its sides is always in a consistent relationship with one another. In the below-given 30-60-90 triangle ABC, ∠ C = 30°,∠ A = 60°, and ∠ B = 90°. We can understand the relationship between each of the sides from the below definitions: The side that is opposite to the 30° angle, AB = y will always be the smallest since 30° is the smallest angle in this triangle The side that is opposite to the 60° angle, BC = y × √3 = y√3 will be the medium length because 60° is the mid-sized degree angle in this triangle On the side that is opposite to the 90° angle, the hypotenuse AC = 2y will be the largest side because 90° is the largest angle. The sides of a 30-60-90 triangle are always in the ratio of 1:√3: 2. This is also known as the 30-60-90 triangle formula for sides y: y√3: 2y. Let us learn the derivation of this ratio in the 30-60-90 triangle proof section. This formula can be verified using the Pythagoras theorem. Consider some of the examples of a 30-60-90 degree triangle with these side lengths: Here, in the 30-60-90 triangle DEF ∠ F = 30°,∠ D = 60°, and ∠ E = 90° The side opposite to the 30° angle, DE = y = 2 The side opposite to the 60° angle, BC = y√3 = 2√3 The side opposite to the 90° angle, the hypotenuse AC = 2y = 2 × 2 = 4 Here, in the 30-60-90 triangle PQR ∠ R = 30°,∠ P = 60°, and ∠ Q = 90° The side opposite to the 30° angle, AB = y = 7 The side opposite to the 60° angle, BC = y√3 = 7√3 The side opposite to the 90° angle, the hypotenuse AC = 2y = 2 × 7 = 14 30-60-90-Triangle Theorem The statement of the 30-60-90-Triangle Theorem is given as, Statement: The length of the hypotenuse is twice the length of the shortest side and the length of the other side is √3 times the length of the shortest side in a 30-60-90-Triangle. 30-60-90-Triangle Formula The above theorem can be written mathematically as the 30-60-90-Triangle Formula as 1:√3: 2 which is the ratio of the three sides of the 30-60-90-Triangle. Another formula for this special triangle 1:2:3 which is the ratio of the three angles of the 30-60-90-Triangle. 30-60-90-Triangle Proof Let's consider an equilateral triangle ABC with a side length equal to 'a'. Now, draw a perpendicular from vertex A to side BC at point D of the triangle ABC. The perpendicular in an equilateral triangle bisects the other side. Triangle ABD and ADC are two 30-60-90 triangles. Both the triangles are similar and right-angled triangles. Hence, we can apply the Pythagoras theorem to find the length AD. (AB)2 = (AD)2 + (BD)2 a2 = (AD)2 + (a/2)2 a2 - (a/2)2 = (AD)2 3a2/4 = (AD)2 (a√3)/2 = AD AD = (a√3)/2 BD = a/2 AB = a These sides also follow the same ratio a/2 : (a√3)/2: a Multiply by 2 and divide by 'a', (2a)/(2a) : (2a√3)/(2a): (2a/a) We get 1:√3:2. This is the 30-60-90 triangle theorem. 30-60-90 Triangle Rule In a 30-60-90 triangle, the measure of any of the three sides can be found out by knowing the measure of at least one side in the triangle. This is called the 30-60-90 triangle rule. The below-given table shows how to find the sides of a 30-60-90 triangle using the 30-60-90 triangle rule: Base is given Perpendicular is given Hypotenuse is given The Base BC of the triangle is assumed to be 'a'. The perpendicular DE of the triangle is assumed to be 'a'. The hypotenuse PR of the triangle is assumed to be 'a'. The perpendicular of the triangle ABC is AB = (a /√3) The hypotenuse of the triangle ABC is AC = (2a)/√3 The base of the triangle DEF is EF = √3a. The hypotenuse of the triangle DEF is DF = 2a. The base of the triangle PQR is QR = (√3a)/2. The perpendicular of the triangle PQR is PQ = (a/2). Area of a 30-60-90 Triangle The formula to calculate the area of a triangle is = (1/2) × base × height. In a right-angled triangle, the height is the perpendicular of the triangle. Thus, the formula to calculate the area of a right-angle triangle is = (1/2) × base × perpendicular Let's learn how to apply this formula to find the area of the 30-60-90 triangle. Base BC of the triangle is assumed to be 'a', and the hypotenuse of the triangle ABC is AC. We have learned in the previous section how to find the hypotenuse when the base is given. Let's apply the formula we have learned. Thus, perpendicular of the triangle = a/√3 Area of the triangle = (1/2) × base × perpendicular Area = 1/2 × a × a/√3 Therefore, the area of the 30-60-90 triangle when the base (side of middle length) is given as 'a' is: a2/(2√3) Related Articles Important Notes on 30-60-90 Triangle Here is a list of a few points that should be remembered while studying 30-60-90 triangles: The 30-60-90 triangle is called a special right triangle as the angles of this triangle are in a unique ratio of 1:2:3 and the sides are in the ratio 1:√3: 2 A 30-60-90 triangle is a special right triangle that always has angles of measure 30°, 60°, and 90° All the sides of a 30-60-90 triangle can be calculated if any one side is given. This is called the 30-60-90 triangle rule. Example 1: Find the length of the hypotenuse of a right-angle triangle if the other two sides are 8 and 8√ 3 units. Solution: First, let's check the ratio to verify if it is suitable for a 30-60-90 triangle. The ratio of the two sides = 8:8√3 = 1:√3 This indicates that the triangle is a 30-60-90 triangle. We know that the hypotenuse is 2 times the smallest side. Thus, the hypotenuse is 2 × 8 = 16 units Answer: Hypotenuse = 16 units Example 2: A triangle has sides 2√2, 2√6, and 2√8. Find the angles of this triangle. Solution: The sides of the triangle are 2√2, 2√6, and 2√8. First, let's check whether the sides are following the 30-60-90 triangle rule. 2√2: 2√6: 2√8 can be re-written as 2√2: 2√2 × √3: 2 × 2√2 If we divide the ratio by 2√2, we get 1:√3: 2 These sides are following the 30-60-90 triangle rule. We know that the angles of a 30-60-90 triangle are 30°,60°, and 90°. Answer: The angles of the triangle are 30°,60°, and 90°. Example 3: Verify if the triangle is a 30-60-90 triangle if the sides of the triangle are 4 units, √48 units, and 8 units. Solution: To verify if the triangle is a 30-60-90 triangle, we will check the ratio of the sides. We have 4 : √48 units : 8 = 4 : 4√3 : 4 × 2 = 1 : √3 : 2 Using the 30-60-90 triangle theorem, the sides are in the ratio 1 : √3 : 2. So the triangle is a 30-60-90 triangle. Answer: The given triangle is a 30-60-90 triangle. Show Answer > go to slidego to slidego to slide Breakdown tough concepts through simple visuals. Math will no longer be a tough subject, especially when you understand the concepts through visualizations with Cuemath. Book a Free Trial Class FAQs on 30-60-90 Triangle The 30-60-90 triangle is called a special right triangle as the angles of this triangle are in a unique ratio of 1:2:3. A 30-60-90 triangle is a special right triangle that always has angles of measure 30°, 60°, and 90°. What Is the Perimeter of a 30-60-90 Triangle? The perimeter of a 30 60 90 triangle with the smallest side equal to a is the sum of all three sides. The other two sides are a√3 and 2a. The perimeter of the triangle is a+a√3+2a = 3a+a√3 = a√3(1+√3). Are There Any Tips for Remembering the 30-60-90 Triangle Rules? This method can be used to remember the 30-60-90 triangle rule. One can remember it as 1, 3, 2; it can resemble the ratio of the sides, all one needs to remember is that the middle term is √3 What Are the Side Lengths of a 30-60-90 Triangle? The sides of a 30-60-90 triangle have a set pattern. The side that is opposite to the 30° angle, 'y' will always be the smallest since 30° is the smallest angle in this triangle. The side that is opposite to the 60° angle, y√3 will be the medium length because 60° is the mid-sized degree angle in this triangle. The side that is opposite to the 90° angle, 2y will be the largest side because 90° is the largest angle. What Are the Rules for a 45-45-90 Triangle? A 45-45-90 triangle has a right angle and two 45 degree angles. The two sides of a 45-45-90 triangle are always equal and the hypotenuse of the triangle is always opposite to the right angle. What Are Some Similarities Between 30-60-90 Triangles and 45-45-90 Triangles? These are some of the similarities between 30-60-90 triangle and 45-45-90 triangle: both are not acute triangles, both are right-angle triangles, both are not obtuse triangles, the square of the hypotenuse equals the sum of the squares of the other two sides for both triangles, and the sum of the interior angles of both are 180°. Which Leg is the Long Leg in the 30-60-90 Triangle? The long leg of a 30-60-90 Triangle is the leg whose length is greater than the shortest leg and ess than the hypotenuse. The length of the long leg is equal to √3 times the length of the shortest leg. home / geometry / triangle / 30-60-90 triangleA 30-60-90 triangle is a right triangle having interior angles measuring 30°, 60°, and 90°. The figure below shows a 30-60-90 triangle example. What is a 30 60 90 triangle A 30-60-90 triangle is a type of right triangle. It is considered a special right triangle because it has predictable and consistent sides and angle measures, which enables us to use shortcuts to determine all the sides and angles of the triangle given enough information. This is useful in both geometry and trigonometry. In the case of a 30-60-90 triangle, only 1 side needs to be known in order to find all the lengths of the other sides. How to know if a triangle is 30 60 90 To know whether a triangle is a 30-60-90 triangle, we only need to know two of the angles. Since the sum of the internal angles of a triangle is 180°, if we know that the angles of a triangle are 30 60, 30 90, or 60 90, then the triangle is guaranteed to be a 30-60-90 triangle. 30 60 90 triangle properties The following are some properties of a 30-60-90 triangle. The shortest side of a 30 60 90 triangle is opposite the 30° angle; the longer side is opposite the 60° angle; the hypotenuse, the longest side, is opposite the 90° angle. The 3 sides of a 30 60 90 triangle always exist in the same ratio. All 30 60 90 triangles are similar; in other words, 30 60 90 triangles only vary in size. They share the same ratios and angles, as depicted in the figure below. How to solve a 30 60 90 triangle To solve a 30-60-90 triangle we use the 30 60 90 triangle rule, also referred to as the 30 60 90 triangle theorem. 30 60 90 triangle rule Given a single side of a 30-60-90 triangle, it is possible to find all the other sides of the triangle since the ratio always remains the same. The ratio of the side lengths of a 30-60-90 triangle are: The leg opposite the 30° angle (the shortest side) is the length of the hypotenuse (the side opposite the 90° angle). The leg opposite the 60° angle is of the length of the hypotenuse. The hypotenuse is twice the length of the shortest side. Thus, given a 30 60 90 triangle and one side of the triangle, we can find the other two sides as follows: Given the length of the hypotenuse, divide the hypotenuse by 2 to find the shorter leg. Then, multiply the shorter leg by to find the longer leg. Given the length of the longer leg, divide by to find the shorter leg. Then multiply the shorter leg by 2 to find the hypotenuse. Given the shorter leg, multiply by 2 to find the hypotenuse and multiply by to find the longer side. Below are some examples of how to find the sides of a 30-60-90 triangle. Examples Find the lengths of the other sides of the 30 60 90 triangle given either the hypotenuse (c), longer side (b), or short side (a); refer to the triangle below. 1. Find a and b given c = 24. Divide the hypotenuse by 2 to find a: Then, multiply a by to find b: 2. Find b and c given a = 8. Multiply a by 2 to find the hypotenuse: Multiply a by to find the longer side: 3. Find a and c given b = 15. Divide by to find a: Multiply a by 2 to find the hypotenuse: 30 60 90 triangle in trigonometry In the study of trigonometry, the 30-60-90 triangle is considered a special triangle. Knowing the ratio of the sides of a 30-60-90 triangle allows us to find the exact values of the three trigonometric functions sine, cosine, and tangent for the angles 30° and 60°. For example, sin(30°), read as the sine of 30 degrees, is the ratio of the side opposite the 30° angle of a right triangle, to its hypotenuse. Using the side lengths for the 30-60-90 triangle above: Similarly, we can find that: 30 60 90 triangle theorem proof The ratios of the sides can be calculated using two congruent 30-60-90 triangles. As shown in the figure above, two congruent 30-60-90 triangles, △ACD and △BCD, share a side along their longer leg. Since ∠BCD = ∠ACD = 30°, ∠BCA = 60°. Also ∠CAD = ∠CBD = 60°, therefore △ABC is an equilateral triangle. Given that the length of one side of the equilateral triangle is s, AB = AC = BC = s and . Using the Pythagorean Theorem for either triangle, So: Area of a 30 60 90 triangle The area of a 30-60-90 triangle can be found using a variation of the standard formula . Since we know the ratios of the length of the sides of a 30-60-90 triangle, we can find the area of a 30-60-90 triangle using the formula: Perimeter of a 30 60 90 triangle The perimeter of a 30-60-90 triangle can be simplified since the ratio of the sides is known. The perimeter of a 30-60-90 triangle can be found using the perimeter formula: where a is the length of the short side of the triangle. Page 2home / geometry / triangle / 45-45-90 triangleA 45-45-90 triangle is a right triangle having interior angles measuring 45°, 45°, and 90°. A 45-45-90 triangle is also an isosceles triangle, which means its two legs are equal in length. Similarity All 45-45-90 triangles are similar. Line segments DE and FG are perpendicular to side AB of the 45-45-90 triangle, ABC. Triangles ADE and AFG are also 45-45-90 triangles so, △ABC~△ADE~△AFG. 45-45-90 triangle side lengths The ratio of the side lengths of a 45-45-90 triangle are: The legs opposite the 45° angles (the shorter sides) are of the length of the hypotenuse (the side opposite the 90° angle) The hypotenuse is times the length of either leg. Since a 45-45-90 triangle is also an isosceles triangle, the two legs are equal in measure. Assuming x is the length of the leg and b is the length of the hypotenuse and using the Pythagorean Theorem: x2 + x2 = b2 Thus, the ratio of the side lengths of a 45-45-90 triangle are or respectively. Example: Find the lengths of the legs for △PQR below. △PQR is a 45-45-90 triangle since ∠P≅∠R and ∠Q=90°. Sides PQ and QR are of the length of PR: 45-45-90 triangle in trigonometry In the study of trigonometry, the 45-45-90 triangle is considered a special triangle. Knowing the ratio of the sides of a 45-45-90 triangle allows us to find the exact values of the three trigonometric functions sine, cosine, and tangent for the angle 45°. For example, sin(45°), read as the sine of 45 degrees, is the ratio of the side opposite the 45° angle of a right triangle, to its hypotenuse. Using the side lengths , for the 45-45-90 triangle above: Similarly: tan(45°) = 1 A 30-60-90 triangle is a special right triangle whose three angles are 30°, 60°, and 90°. The triangle is special because its side lengths are in the ratio of 1: √3: 2 (x: x√3: 2x for shorter side: longer side: hypotenuse). 30 60 90 Triangle Since a 30-60-90 triangle is a right triangle, the Pythagoras formula a2 + b2 = c2, where a = longer side, b = shorter side, and c = hypotenuse is also applicable. For example, the hypotenuse can be obtained when the two other sides are known as shown below. ⇒ c2 = x2 + (x√3)2 ⇒ c2 = x2 + (x√3) (x√3) ⇒ c2 = x2 + 3x2 ⇒ c2 = 4x2 Squaring both sides, we get, √c2 = √4x2 c = 2x The angles are in the ratio 1: 2: 3, which are in arithmetic progressionThe sides are in the ratio 1: √3: 2 (x: √3x: 2x)The side opposite the 30° angle is the shorter side, denote by xThe side opposite the 60° angle is the longer side, denote by x√3The side opposite the 90° angle is the hypotenuse, denote by 2xAll 30-60-90 triangles are similar trianglesTwo 30-60-90 triangles sharing the longer side form an equilateral triangle From the above properties, we get some basic rules applicable in all 30-60-90 triangles. The three side lengths are always in the ratio of 1: √3: 2 and the shortest side is always the smallest angle (30°), while the longest side is always opposite the largest angles (90°). These rules are useful for solving the 30-60-90 theorem that we will deal with in the next section. 30 60 90 Triangle Theorem Thus, the properties 2, 3, 4, and 5 are collectively called the 30-60-90 triangle theorem, which is summarized below: The hypotenuse is twice the length of the short legThe length of the longer side is √3 times the shorter side To prove 30-60-90 Triangle Theorem To prove:Let △ABC be an equilateral triangle with each side length equal to x.Proof:Given: △ABC is an equilateral triangle with side length ‘x’Proof:A perpendicular line is drawn from vertex A to side BC that meets at point D such that it bisects the side BC. The two triangles formed, △ABD and △ADC are similar.Since, both the triangles are right triangles; here we will use the Pythagorean Theorem to find the length of AD.Thus,AB2 = AD2 + BD2x2 = AD2 +(x/2)2x2 – (x/2)2 = AD2AD2 = 3x2/4 =x√3/2BD = x/2AB = xThus, the three sides are in the ratio of: x/2: x√3/2: xMultiplying by 2, we get,1: √3: 2Hence proved that the given △ABC is a 30-60-90 Triangle. Given the length of one side of a triangle, we can find the other side(s) without using long-step methods such as Pythagorean Theorem and trigonometric functions. Solving a 30-60-90 triangle can have four possibilities: Possibility 1: When the shorter side is known, we can find the longer side by multiplying the shorter side by √3. The hypotenuse can then be obtained by Pythagoras Theorem. Thus, Longer side = shorter side × √3 Possibility 2: When the longer side is known, we can find the shorter side by dividing the longer side by √3. The hypotenuse can then be obtained by Pythagoras Theorem. Thus, Shorter side = longer side/√3 Possibility 3: When the shorter side is known, we can find the hypotenuse by multiplying the shorter side by 2. The longer side can then be obtained by Pythagoras Theorem. Thus, Hypotenuse = shorter side × 2 Possibility 4: When the hypotenuse is known, we can find the shorter side by dividing the hypotenuse by 2. The longer side can then be obtained by Pythagoras Theorem. Thus, Shorter side = hypotenuse/2 The formulas of a 30-60-90 triangle when the length of the shorter side is x units are given below: 30 60 90 Triangle Formula Let us solve some examples to understand the concepts better. A right triangle whose one angle is 60 degrees has the longer side of 12√3 cm. Find the length of its shorter side and the hypotenuse.Solution:Since, the given triangle is a 30-60-90 triangle,Its side lengths = x: x√3: 2x, here x√3 = longer side = 12√3 cmThus,x√3 =12√3Squaring both sides we get,⇒ (x√3)2 = (12√3)2 cm⇒ 3x2 = 144 x 3⇒ x2 = 144⇒ x = 12 cmHypotenuse = 2x = 2 x 12 = 24 cm Hence the shorter side is 12cm and hypotenuse is 24 cm. A right triangle whose one angle is 60 degrees has the longer side of 12√3 cm. Find the length of its shorter side and the hypotenuse.Solution:Since, the given triangle is a 30-60-90 triangle,Ratios of a 30-60-90 triangle side lengths = x: x√3: 2x, here x√3 = longer side = 12√3 cmThus,x√3 =12√3Squaring both sides we get,⇒ (x√3)2 = (12√3)2 cm⇒ 3x2 = 144 x 3⇒ x2 = 144⇒ x = 12 cmHypotenuse = 2x = 2 x 12 = 24 cm Hence the shorter side is 12cm and hypotenuse is 24 cm. The diagonal of a right triangle is 14 cm, find the lengths of the other two sides of the triangle given that one of its angles is 30 degrees.Solution:Since, the given triangle is a 30-60-90 triangle,Ratios of a 30-60-90 triangle side lengths = x: x√3: 2x, here 2x = diagonal = hypotenuse = 12√3 cmThus,⇒ 2x = 14 cm⇒ x = 7 cmSubstituting the value of x, we get,Longer side = x√3 = 7√3 cmHence, the length of the longer side is 7√3 cm. Find the value of y and z in the given diagram.Solution:Since, the given triangle is a 30-60-90 triangle,The side measuring 18m is the shorter side because it is opposite the 30-degree angle.Ratios of a 30-60-90 triangle side lengths = x: x√3: 2x, here x = 18mSubstituting the value of x, we get,Given,x√3 = yLonger side = y = 18√3mGiven,2x = zHypotenuse = z = 2 x 18 = 36mHence, the length of the hypotenuse is 36m. If one angle of a right triangle is 30° and the measure of the shortest side is 11cm. Find the measure of the remaining two sides.Solution:Since, the given triangle is a 30-60-90 triangle,Ratios of a 30-60-90 triangle side lengths = x: x√3: 2x, here x = 7cmSubstituting the value of x, we get,Longer side = x√3 = 7√3cmHypotenuse = 2x = 2 x 7 = 14cmHence, the length of the other two sides is 7√3cm and 14cm. A ramp making an angle of 30 degrees with the ground is used to offload a lorry that is 8 m high. Calculate the length of the ramp.Solution:Since, the ramp makes a 30-60-90 triangle with the ground,Here,Shorter side = x = 8mSince, the length of the ramp is the hypotenuse of the given 30-60-90 triangle,2x = 2 x 8 = 16mHence the length of the ramp is 16m. Last modified on August 3rd, 2023 Written by January 11, 2023Fact-checked by Paul MazzolaDefinitionRatioTheoremHow to solveExamplesA 30-60-90 triangle is a right triangle where the three interior angles measure 30°, 60°, and 90°. Right triangles with 30-60-90 interior angles are known as special right triangles.Special triangles in geometry because of the powerful relationships that unfold when studying their angles and sides.What is a 30-60-90 triangleIn all triangles, the relationships between angles and their opposite sides are easy to understand. The greater the angle, the longer the opposite side.Triangle relationships between angles and opposite sidesThis means, of the three interior angles, the largest interior angle is opposite the longest of the three sides, and the smallest angle will be opposite the shortest side.In a right triangle, recall that the side opposite the right angle (the largest angle) is called the hypotenuse (the longest side, and the other two sides are called legs.Get free estimates from geometry tutors near you.30-60-90 triangle ratioA 30-60-90 degree triangle is a special right triangle, so it's side lengths are always consistent with each other. The ratio of the sides follow the 30-60-90 triangle ratio:Short side (opposite the 30 degree angle) = xHypotenuse (opposite the 90 degree angle) = 2xLong side (opposite the 60 degree angle) = x√330-60-90 triangle ratio and theoremThese three special properties can be considered the 30-60-90 triangle theorem and are unique to these special right triangles:The hypotenuse (the triangle's longest side) is always twice the length of the short legThe length of the longer leg is the short leg's length times √3If you know the length of any one side of a 30-60-90 triangle, you can find the missing side lengthsOther interesting properties of 30-60-90 triangles are:All 30-60-90 triangles are similarTwo 30-60-90 triangles sharing a long leg form an equilateral triangleOther interesting properties of special right trianglesEducation is knowing that 30-60-90 triangles have three properties laid out in the theorem. Wisdom is knowing what to do with that knowledge. Suppose you have a 30-60-90 triangle:How to solve a 30-60-90 triangleWe know that the hypotenuse of this triangle is twice the length of the short leg:We also know that the long leg is the short leg multiplied times the square root of 3:We set up our special 30-60-90 to showcase the simplicity of finding the length of the three sides. Try figuring this one out:30-60-90 triangle example problemThe long leg is the short leg times √3, so can you calculate the short leg's length? Did you say 5?The length of the hypotenuse is always twice the short leg's length. Did you get 10?You can create your own 30-60-90 Triangle formula using the known information in your problem and the following rules. This table of 30-60-90 triangle rules to help you find missing side lengths.30-60-90 triangle rules30-60-90 triangle rules If you know... Then... To get... Hypotenuse Divide by 2 Short leg Short leg Multiply by 2 Hypotenuse Short leg Multiply by √3 Long leg Long leg Divide by √3 Short leg When working with 30-60-90 triangles, you may be tempted to force a relationship between the hypotenuse and the long leg. That relationship is challenging because of the square root of 3.Work carefully, concentrating on the relationship between the hypotenuse and short leg, then short leg and long leg.You will notice our examples so far only provided information that would "plugin" easily using our three properties. Sometimes the geometry is not so easy.30-60-90 triangle rulesWhat if the long leg is labeled with a simple, whole number?You leap into the problem since getting the short leg is simply a matter of dividing the long leg by the square root of 3, then doubling that to get the hypotenuse.But you cannot leave the problem like this:The rules of mathematics do not permit a radical in the denominator, so you must rationalize the fraction. Multiply both numerator and denominator times √3:This simplifies to:Unless your directions are to provide a decimal answer, this can be your final answer for the length of the short side. Doubling this gives 18 √3 for the hypotenuse.Another warning flag with 30-60-90 triangles is that you can become so engrossed in the three properties that you lose sight of the triangle itself. It is still a triangle, so its interior angles must add to 180°, and its three sides must still adhere to the Pythagorean Theorem:You can use the Pythagorean Theorem to check your work or to jump-start a solution.Get free estimates from geometry tutors near you.30-60-90 triangle and Pythagorean theoremA right triangle has a short side with a length of 14 meters with the opposite angle measuring 30°. What are the other two lengths?30-60-90 triangle - finding missing lengthsWe know immediately that the triangle is a 30-60-90, since the two identified angles sum to 120°:The missing angle measures 60°. It follows that the hypotenuse is 28 m, and the long leg is 14 m * √3.What is you have a triangle with the hypotenuse labeled 2,020 mm, the short leg labeled 1,010 mm, and the long leg labeled 1,010√3.Your knowledge of the 30-60-90 triangle will help you recognize this immediately. You can confidently label the three interior angles because you see the relationships between the hypotenuse and short leg and the short leg and long leg.Here is a more elaborate problem. What is the length of the shorter leg, line segment MH?:Special triangle example problemDid you say 50 inches? This is really two 30-60-90 triangles, which means hypotenuse MA is also 100 inches, which means the shortest leg MH is 50 inches. Acute, obtuse, isosceles, equilateral...When it comes to triangles, there are many different varieties, but only a choice few that are "special." These special triangles have sides and angles which are consistent and predictable and can be used to shortcut your way through your geometry or trigonometry problems. And a 30-60-90 triangle—pronounced "thirty sixty ninety"—happens to be a very special type of triangle indeed. In this guide, we'll walk you through what a 30-60-90 triangle is, why it works, and when (and how) to use your knowledge of it. So let's get to it! What Is a 30-60-90 Triangle? A 30-60-90 triangle is a special right triangle (a right triangle being any triangle that contains a 90 degree angle) that always has degree angles of 30 degrees, 60 degrees, and 90 degrees. Because it is a special triangle, it also has side length values which are always in a consistent relationship with one another. The basic 30-60-90 triangle ratio is: Side opposite the 30° angle: x Side opposite the 60° angle: $x * √3$ Side opposite the 90° angle: $2x$ For example, a 30-60-90 degree triangle could have side lengths of: 2, 2√3, 4 7, 7√3, 14 √3, 3, 2√3 (Why is the longer leg 3? In this triangle, the shortest leg (x) is $√3$, so for the longer leg, $x√3 = √3 * √3 = √9 = 3$. And the hypotenuse is 2 times the shortest leg, or $2√3$) And so on. The side opposite the 30° angle is always the smallest, because 30 degrees is the smallest angle. The side opposite the 60° angle will be the middle length, because 60 degrees is the mid-sized degree angle in this triangle. And, finally, the side opposite the 90° angle will always be the largest side (the hypotenuse) because 90 degrees is the largest angle. Though it may look similar to other types of right triangles, the reason a 30-60-90 triangle is so special is that you only need three pieces of information in order to find every other measurement. So long as you know the value of two angle measures and one side length (doesn't matter which side), you know everything you need to know about your triangle. For example, we can use the 30-60-90 triangle formula to fill in all the remaining information blanks of the triangles below. Example 1 We can see that this is a right triangle in which the hypotenuse is twice the length of one of the legs. This means this must be a 30-60-90 triangle and the smaller given side is opposite the 30°. The longer leg must, therefore, be opposite the 60° angle and measure $6 * √3$, or $6√3$. Example 2 We can see that this must be a 30-60-90 triangle because we can see that this is a right triangle with one given measurement, 30°. The unmarked angle must then be 60°. Since 18 is the measure opposite the 60° angle, it must be equal to $x√3$. The shortest leg must then measure $18/√3$. (Note that the leg length will actually be $18/{√3} * {√3}/{√3} = {18√3}/3 = 6√3$ because a denominator cannot contain a radical/square root). And the hypotenuse will be $2(18/√3)$ (Note that, again, you cannot have a radical in the denominator, so the final answer will really be 2 times the leg length of $6√3$ => $12√3$). Example 3 Again, we are given two angle measurements (90° and 60°), so the third measure will be 30°. Because this is a 30-60-90 triangle and the hypotenuse is 30, the shortest leg will equal 15 and the longer leg will equal 15√3. No need to consult the magic eight ball—these rules always work. Why It Works: 30-60-90 Triangle Theorem Proof But why does this special triangle work the way it does? How do we know these rules are legit? Let's walk through exactly how the 30-60-90 triangle theorem works and prove why these side lengths will always be consistent. First, let's forget about right triangles for a second and look at an equilateral triangle. An equilateral triangle is a triangle that has all equal sides and all equal angles. Because a triangle's interior angles always add up to 180° and $180/3 = 60$, an equilateral triangle will always have three 60° angles. Now let's drop down a height from the topmost angle to the base of the triangle. We've now created two right angles and two congruent (equal) triangles. How do we know they're equal triangles? Because we dropped a height from an equilateral triangle, we've split the base exactly in half. The new triangles also share one side length (the height), and they each have the same hypotenuse length. Because they share three side lengths in common (SSS), this means the triangles are congruent. Note: not only are the two triangles congruent based on the principles of side-side-side lengths, or SSS, but also based on side-angle-side measures (SAS), angle-angle-side (AAS), and angle-side-angle (ASA). Basically? They're most definitely congruent. Now that we've proven the congruencies of the two new triangles, we can see that the top angles must each be equal to 30 degrees (because each triangle already has angles of 90° and 60° and must add up to 180°). This means we have made two 30-60-90 triangles. And because we know that we cut the base of the equilateral triangle in half, we can see that the side opposite the 30° angle (the shortest side) of each of our 30-60-90 triangles is exactly half the length of the hypotenuse. So let us call our original side length x and our bisected length $x/2$. Now all that leaves us to do is to find our mid-side length that the two triangles share. To do this, we can simply use the Pythagorean theorem. $a^2 + b^2 = c^2$ $(x/2)^2 + b^2 = x^2$ $b^2 = x^2 - ({x^2}/4)$ $b^2 = {4x^2}/4 - {x^2}/4$ $b^2 = {3x^2}/4$ $b = {√3x}/2$ So we're left with: $x/2, {x√3}/2, x$ Now let's multiply each measure by 2, just to make life easier and avoid all the fractions. That way, we're left with: x, $x√3$, $2x$ We can see, therefore, that a 30-60-90 triangle will always have consistent side lengths of x, $x√3$, and $2x$ (or $x/2$, ${√3x}/2$, and x). Luckily for us, we can prove 30-60-90 triangle rules true without all of...this. When to Use 30-60-90 Triangle Rules Knowing the 30-60-90 triangle rules will be able to save you time and energy on a multitude of different math problems, namely a wide variety of geometry and trigonometry problems. Geometry Proper understanding of the 30-60-90 triangles will allow you to solve geometry questions that would either be impossible to solve without knowing these ratio rules, or at the very least, would take considerable time and effort to solve the "long way." With the special triangle ratios, you can figure out missing triangle heights or leg lengths (without having to use the Pythagorean theorem), find the area of a triangle by using missing height or base length information, and quickly calculate perimeters. Any time you need speed to answer a question, remembering shortcuts like your 30-60-90 rules will come in handy. Trigonometry Memorizing and understanding the 30-60-90 triangle ratio will also allow you to solve many trigonometry problems without either the need for a calculator or the need to approximate your answers in decimal form. A 30-60-90 triangle has fairly simple sines, cosines, and tangents for each angle (and these measurements will always be consistent). Sine of 30° will always be $1/2$. Cosine of 60° will always be $1/2$. Though the other sines, cosines, and tangents are fairly simple, these are the two that are the easiest to memorize and are likely to show up on tests. So knowing these rules will allow you to find these trigonometry measurements as quickly as possible. Tips for Remembering the 30-60-90 Rules You know these 30-60-90 ratio rules are useful, but how do you keep the information in your head? Remembering the 30-60-90 triangle rules is a matter of remembering the ratio of 1: √3 : 2, and knowing that the shortest side length is always opposite the shortest angle (30°) and the longest side length is always opposite the largest angle (90°). Some people memorize the ratio by thinking, "$\bi x$, $\bo 2 \bi x$, $\bi x \bo √ \bo3$," because the "1, 2, 3" succession is typically easy to remember. The one precaution to using this technique is to remember that the longest side is actually the $2x$, not the x times $√3$. Another way to remember your ratios is to use a mnemonic wordplay on the 1: root 3: 2 ratio in their proper order. For example, "Jackie Mitchell struck out Lou Gehrig and 'won Ruthy too,'": one, root three, two. (And it's a true baseball history fact to boot!) Play around with your own mnemonic devices if these don't appeal to you—sing the ratio to a song, find your own "one, root three, two" phrases, or come up with a ratio poem. You can even just remember that a 30-60-90 triangle is half an equilateral and figure out the measurements from there if you don't like memorizing them. However it makes sense to you to remember these 30-60-90 rules, keep those ratios your head for your future geometry and trigonometry questions. Memorization is your friend, however you can make it happen. Example 30-60-90 Questions Now that we've looked at the hows and whys of 30-60-90 triangles, let's work through some practice problems. Geometry A construction worker leans a 40-foot ladder up against the side of a building at an angle of 30 degrees off the ground. The ground is level and the side of the building is perpendicular to the ground. How far up the building does the ladder reach, to the nearest foot? Without knowing our 30-60-90 special triangle rules, we would have to use trigonometry and a calculator to find the solution to this problem, since we only have one side measurement of a triangle. But because we know that this is a special triangle, we can find the answer in just seconds. If the building and the ground are perpendicular to one another, that must mean the building and the ground form a right (90°) angle. It's also a given that the ladder meets the ground at a 30° angle. We can therefore see that the remaining angle must be 60°, which makes this a 30-60-90 triangle. Now we know that the hypotenuse (longest side) of this 30-60-90 is 40 feet, which means that the shortest side will be half that length. (Remember that the longest side is always twice—$2x$—as long as the shortest side.) Because the shortest side is opposite the 30° angle, and that angle is the degree measure of the ladder from the ground, that means that the top of the ladder hits the building 20 feet off the ground. Our final answer is 20 feet. Trigonometry If, in a right triangle, sin Θ = $1/2$ and the shortest leg length is 8. What is the length of the missing side that is NOT the hypotenuse? Because you know your 30-60-90 rules, you can solve this problem without the need for either the pythagorean theorem or a calculator. We were told that this is a right triangle, and we know from our special right triangle rules that sine 30° = $1/2$. The missing angle must, therefore, be 60 degrees, which makes this a 30-60-90 triangle. And because this is a 30-60-90 triangle, and we were told that the shortest side is 8, the hypotenuse must be 16 and the missing side must be $8 * √3$, or $8√3$. Our final answer is 8√3. The Take-Aways Remembering the rules for 30-60-90 triangles will help you to shortcut your way through a variety of math problems. But do keep in mind that, while knowing these rules is a handy tool to keep in your belt, you can still solve most problems without them. Keep track of the rules of x, $x√3$, $2x$ and 30-60-90 in whatever way makes sense to you and try to keep them straight if you can, but don't panic if your mind blanks out when it's crunch time. Either way, you've got this. And, if you need more practice, go ahead and check out this 30-60-90 triangle quiz. Happy test-taking!