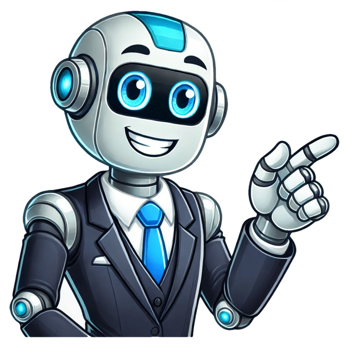

We can have two types of binomials cubed, a difference or a sum. The sum of cubes is an expression with the general form $latex {{a}^3}+{{b}^3}$ and a difference of cubes is an expression with the general form $latex {{a}^3}-{{b}^3}$. To factor binomials cubed, we can follow the following steps: Step 1: Factor the common factor of the terms if it exists to obtain a simpler expression. We must not forget to include the common factor in the final answer. Step 2: We have to rewrite the expression as a sum or difference of two perfect cubes. Step 3: We can write the answer using the following sentences: a) "Write what you see" b) "Square – Multiply – Square" c) If it is a sum of cubes, we have the signs "Positive, Negative, Positive" and if it is a difference of cubes, we have the signs "Negative, Positive, Positive" Step 4: We join the resulting parts to obtain the final factored expression. The following examples of factoring binomials cubed apply the solving process detailed above. 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If we square the second term, 5, we have 25. $latex 4{{x}^2},~~10x,~~25$ c) For a sum of cubes, we must use the signs "Positive, Negative, Positive". Step 4: The factored expression is: $latex (2x+5)(4{{x}^2}-10x+25)$ Factor the difference of cubes $latex 27{{x}^3}-216{{y}^3}$. Step 1: We cannot factor initially because we do not have common factors. Step 2: We write the expression as a difference of two perfect cubes: ⇒ $latex {{(3x)}^3}-{{(6y)}^3}$ Step 3: a) By removing parentheses and cubes, we have the expression: $latex 3x-6y$ b) Squaring the first term, 3x, we have $latex 9{{x}^2}$. By multiplying the terms 3x and 6y, we get $latex 18xy$. If we square the second term, 6y, we get $latex 36{{y}^2}$. $latex 9{{x}^2},~~18xy,~~36{{y}^2}$ c) For a difference of cubes, we have the signs "Negative, Positive, Positive". Step 4: The final factored expression is: $latex (3x-6y)(9{{x}^2}+18xy+36{{y}^2})$ Facor the sum of cubes $latex 54{{x}^3}+16{{y}^3}$. 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Step 2: If we rewrite the expression as a difference of two perfect cubes, we have: ⇒ $latex {{(2)}^3}-{{(3xy)}^3}$ Step 3: a) If we remove the parentheses and ignore the cubes, we see the expression: $latex 2-3xy$ b) The first term, 2, when squared is 4. The product of the terms 2 and 3xy is $latex 6xy$. The second term, 3xy squared is $latex 9{{x}^2}{{y}^2}$. $latex 4,~~6xy,~~9{{x}^2}{{y}^2}$ c) For a difference of cubes, the signs are "Negative, Positive, Positive". Step 4: The factored expression is: $latex (2-3xy)(4+6xy+9{{x}^2}{{y}^2})$ Practice what you have learned about factoring binomials cubed with the following problems. If you need help, you can look at the solved examples shown above. Interested in learning more about the factorization of polynomials? Take a look at these pages: .author-box {margin: 70px 0; padding: 30px; background-color: #f9fcff; border-radius: 15px; box-shadow: 0px 0px 10px #ccc; max-width:1100px; margin-left:auto !important; margin-right:0px !important; } .author-box img {margin:auto; border-radius: 50%;} .author-box h3 {margin-top: 20px; font-size:19px;} .author-box p {margin: 10px 0; text-align:left; } .author-box a {display: inline-block; margin-right: 10px; color: black; text-decoration: none;} { "@context": " ", "@type": "Person", "name": "Jefferson Huera Guzman", "image": " , "url": " , "description": "Jefferson is the lead author and administrator of Neurochispas.com.", "sameAs": [" , "], "email": "", "worksFor": { "@type": "Organization", "name": "Interacti Digital LLC"}, "alumniOf": { "@type": "CollegeOrUniversity", "name": "The University of Manchester"}, "knowsAbout": ["Algebra", "Calculus", "Geometry", "Mathematics", "Physics"] } We can have two types of binomials cubed, a difference or a sum. The sum of cubes is an expression with the general form $latex {{a}^3}+{{b}^3}$ and a difference of cubes is an expression with the general form $latex {{a}^3}-{{b}^3}$. To factor binomials cubed, we can follow the following steps: Step 1: Factor the common factor of the terms if it exists to obtain a simpler expression. We must not forget to include the common factor in the final answer. Step 2: We have to rewrite the expression as a sum or difference of two perfect cubes. Step 3: We can write the answer using the following sentences: a) "Write what you see" b) "Square – Multiply – Square" c) If it is a sum of cubes, we have the signs "Positive, Negative, Positive" and if it is a difference of cubes, we have the signs "Negative, Positive, Positive" Step 4: We join the resulting parts to obtain the final factored expression. The following examples of factoring binomials cubed apply the solving process detailed above. 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Step 1: Here, we do have a common factor, 2. We can extract the 2 from both terms: $latex 2(27{{x}^3}+8{{y}^3})$ Step 2: Now, we can write the expression as a sum of two perfect cubes: ⇒ $latex 2({{(3x)}^3}+{{(2y)}^3})$ Step 3: a) If we ignore the parentheses and the cubes, we have: $latex 3x+2y$ b) The first term, 3x, when squared is equal to $latex 9{{x}^2}$. The product of the terms 3x and 2y is $latex 6xy$. The square of the second term, 2y, is $latex 4{{y}^2}$. $latex 9{{x}^2},~~6xy,~~4{{y}^2}$ c) For a sum of cubes, we have the signs "Positive, Negative, Positive". Step 4: We obtained the following factorization: $latex 2(3x+2y)(9{{x}^2}-6xy+4{{y}^2})$ Obtain the factorization of $latex 8-27{{x}^3}{{y}^3}$. Step 1: We have no common factor in the terms. 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Take a look at these pages: .author-box {margin: 70px 0; padding: 30px; background-color: #f9fcff; border-radius: 15px; box-shadow: 0px 0px 10px #ccc; max-width:1100px; margin-left:auto !important; margin-right:0px !important; } .author-box img {margin:auto; border-radius: 50%;} .author-box h3 {margin-top: 20px; font-size:19px;} .author-box p {margin: 10px 0; text-align:left; } .author-box a {display: inline-block; margin-right: 10px; color: black; text-decoration: none;} { "@context": " ", "@type": "Person", "name": "Jefferson Huera Guzman", "image": " , "url": " , "description": "Jefferson is the lead author and administrator of Neurochispas.com.", "sameAs": [" , "], "email": "", "worksFor": { "@type": "Organization", "name": "Interacti Digital LLC"}, "alumniOf": { "@type": "CollegeOrUniversity", "name": "The University of Manchester"}, "knowsAbout": ["Algebra", "Calculus", "Geometry", "Mathematics", "Physics"] } Factoring cubic equations is significantly more challenging than factoring quadratics – there are no guaranteed-to-work methods like guess-and-check and the box method, and the cubic equation, unlike the quadratic equation, is so lengthy and convoluted that it is almost never taught in math classes. Fortunately, there are simple formulas for two types of cubics: the sum of cubes and the difference of cubes. These binomials always factor into the product of a binomial and a trinomial. Take the cube root of the two binomial terms. The cube root of A is the number that, when cubed, is equal to A; for example, the cube root of 27 is 3 because 3 cubed is 27. The cube root of x^3 is simply x. Write the sum of the cube roots of the two terms as the first factor. For example, in the sum of cubes "x^3 + 27," the two cube roots are x and 3, respectively. The first factor is therefore (x + 3). Square the two cube roots to get the first and third term of the second factor. Multiply the two cube roots together to get the second term of the second factor. In the above example, the first and third terms are x^2 and 9, respectively (3 squared is 9). The middle term is 3x. Write out the second factor as the first term minus the second term plus the third term. In the above example, the second factor is (x^2 – 3x + 9). Multiply the two factors together to get the factored form of the binomial: (x + 3)(x^2 – 3x + 9) in the example equation. Take the cube root of the two binomial terms. The cube root of A is the number that, when cubed, is equal to A; for example, the cube root of 27 is 3 because 3 cubed is 27. The cube root of x^3 is simply x. Write the difference of the cube roots of the two terms as the first factor. For example, in the difference of cubes "8x^3 – 8," the two cube roots are 2x and 2, respectively. The first factor is therefore (2x – 2). Square the two cube roots to get the first and third term of the second factor. Multiply the two cube roots together to get the second term of the second factor. In the above example, the first and third terms are 4x^2 and 4, respectively (2 squared is 4). The middle term is 4x. Write out the second factor as the first term minus the second term plus the third term. In the above example, the second factor is (x^2 + 4x + 4). Multiply the two factors together to get the factored form of the binomial: (2x – 2)(4x^2 + 4x + 4) in the example equation. Wallulis, Karl. "How To Factor Binomial Cubes" sciencing.com, . 20 April 2018. APA Wallulis, Karl. (2018, April 20). How To Factor Binomial Cubes. sciencing.com. Retrieved from Chicago Wallulis, Karl. How To Factor Binomial Cubes last modified March 24, 2022. Free Step-by-Step Guide: How to factor a polynomial with a specific number of terms In algebra, a cubic polynomial is an expression made up of four terms that is of the form: Where a, b, c, and d are constants, and x is a variable. Polynomials in this form are called cubic because the highest power of x in the function is 3 (or x cubed).Unlike factoring trinomials, learning how to factor a cubic polynomial can be particularly tricky because using any type of guess-and-check method is extremely difficult. However, you can easily learn how to factor a cubic polynomial by using the grouping method described in this guide.This free Step-by-Step Guide on How to Factorize a Cubic Polynomial will cover the following key topics:While learning how to factor cubic polynomials can be challenging at first, you can develop your skills pretty quickly just by working through practice problems step-by-step until you become more comfortable with factoring cubic polynomials. So, this guide was designed to teach you everything you need to know about how to factor a cubic polynomial. We recommend that you read through this guide from start to finish and work through each example by following along step-by-step. By the end, you will be able to quickly and accurately factorize a cubic polynomial. Figure 01: How to Factorize a Cubic Polynomial As previously mentioned, a cubic polynomial is a math expression that is of the form $ax^3 + bx^2 + cx + d$, where a, b, c, and d are all constants and x is a variable, and typically has four terms. Note that x is not the only letter that can be used as a variable in a cubic polynomial. Also, the number in front of any variable is referred to as a coefficient.Additionally, the terms of a cubic polynomial are the individual "pieces" of the expression, separated by an addition or subtraction sign.For example, the cubic polynomial in Figure 01 above, $x^3 + 3x^2 + 2x + 6$ has four terms:1st Term: $x^3$2nd Term: $3x^2$ 3rd Term: 2x4th term: 6Before you can learn how to factor a cubic polynomial, it is extremely important that you know how to recognize that given polynomial is cubic, so make sure that you deeply understand what a cubic polynomial is before moving forward in this guide. Figure 02: Factoring a Cubic Polynomial In math, the factors of any polynomial represent components or "building blocks" of the polynomial. Whenever you factor a polynomial (cubic or otherwise), you are finding simpler polynomials whose product equals the original polynomial. Each of these simpler polynomials is considered a factor of the original polynomial.For example, the binomial $x^2 - 100$ has two factors (x + 10) and (x-10).Why? Lets take a look at what happens when we find the product of the factors by double distributing:$(x+10)(x-10) = x^2 + 10x - 10x - 100 = x^2 + 0 - 100 = x^2 - 100$Notice that the result was the original polynomial, $x^2 - 100$.Since cubic polynomials (four terms) are more complex than binomials (two terms), their factors will also be a little more complex, but the idea is still the same—factoring a cubic polynomial involves finding simpler polynomials or "building blocks" whose product is the original cubic polynomial.And, to factorize a cubic polynomial, we will be using a strategy called grouping that will allow you to factor any cubic polynomial (assuming that it is factorable at all) using 3 easy steps. So, lets go ahead and work three practice problems to give you some experience with factoring cubic polynomials by grouping.Now that you understand the key terms and the difference between a polynomial with 2 terms, 3 terms, and 4 terms.For factoring each type of polynomial, we will look at two methods: GCF, direct factoring, and, sometimes, a combination of the two. Let's get started! Now, you will learn how to use the follow three steps to factor a cubic polynomial by grouping:Step One: Split the cubic polynomial into two groups of binomials.Step Two: Factor each binomial by pulling out a GCFStep Three: Identify the factors As long as you follow these three steps, you can easily factor a given polynomial, though note that not all cubic polynomials are factorable."We will start by factoring the cubic polynomial shown in Figure 01: $x^3 + 3x^2 + 2x + 6$ Figure 03: We have to find the factors of $x^3 + 3x^2 + 2x + 6$ To factorize this cubic polynomial, we will be applying the previously mentioned 3-step method as follows:Step One: Split the cubic polynomial into groups of two binomials.To factor this cubic polynomial, we will be using the grouping method, where the first step is to split the cubic polynomial in half into two groups. Figure 04: The first step to factoring a cubic polynomial is to split it into groups of two binomials. For Example #1, at the end of the first step, you have split the cubic binomial down the middle to form two groups of binomials:Why are you splitting the cubic polynomial like this? Notice that it is not possible to pull a Greatest Common Factor (GCF) out of the original cubic polynomial $x^3 + 3x^2 + 2x + 6$. The goal of the first step is to create two separate binomials, each with a GCF that can be "pulled out." Figure 05: Make sure that each individual binomial has a GCF before moving onto the next step. Step Two: Factor each binomial by pulling out a GCFAgain, the purpose of the first step is to split the cubic polynomial into two binomials, each with a GCF. Before moving forward, ensure that each individual binomial has a GCF; otherwise, you may need to swap the positions of the middle terms ($3x^2$ and 2x). Swapping these middle terms is not required for this first example; however, we will work through an example later on where this is required.Now, for step two, you can divide the GCF out of each grouping as follows:This process of pulling the GCF out of each binomial is illustrated in Figure 05 below. Figure 06: To factorize a cubic function, split it into two groups and then pull a GCF out of each group. Step Three: Identify the factors After completing the second step, you are left with:Notice that both groups share a common term, which, in this case, is (x+3). This result is expected and is a signal that you are factoring the cubic polynomial correctly. If the groups do not share a common terms, then it is likely that the cubic polynomial is not factorable or that you made a mistake pulling out the GCF.However, since you factored each group and ended up with a common factor of (x+3), you can move on to determining the factors of the cubic polynomial.The illustration in Figure 06 above color-codes how you use the results from step two to determine the factors of the cubic polynomial.You already know that one of the factors is (x+3). To find the other factor, you can simply take the two "outside" terms, in this case, x^2 and +2.$x^2(x + 3) + 2(x+3) \rightarrow (x^2+2)(x+3)$Final Answer: The factors of $x^3 + 3x^2 + 2x + 6$ are (x^2+2) and (x+3)The entire 3-step method that we just used to factor a cubic polynomial by grouping is shown in Figure 07 below: Figure 07: How to factorize a cubic polynomials step-by-step How can you check if your factors are actually correct? You can perform double distribution to multiply the binomials together to see if the result is indeed the cubic polynomial that you started with. If it is, then you know that you have factorized correctly. You can see in Figure 08 below that multiplying the factors together does indeed result in the original cubic polynomial, so you know that your factors are correct:$(x^2+2)(x+3) = x^3 + 3x^2 + 2x + 6$ Figure 08: Check your answer using double distribution Now, lets go ahead and work through another example of how to factor a cubic polynomial. Just like in the first example problem, you can use the 3-steps for factoring a cubic polynomial by grouping as follows: Figure 09: Find the factors of the cubic polynomial $2x^3 - 3x^2 + 18x - 27$ Step One: Split the cubic polynomial into groups of two binomials.After splitting this cubic polynomial, you end up with these two groups: $(2x^3 - 3x^2)$ and (18x-27) Figure 10: Step One: Split the cubic polynomial into two groups Step Two: Factor each binomial by pulling out a GCFNext, divide a GCF out of each group (if possible) as follows:This process of pulling a GCF out of each group is illustrated in Figure 11 below: Figure 11: Factorize a cubic polynomial Step Three: Identify the factors Since both factors have a common term, (2x-3), you know that you have likely factored correctly and you can move onto identifying the factors.Final Answer: (x^2+9) and (2x-3) are the factors of the cubic polynomial $2x^3 - 3x^2 + 18x - 27$.All of the steps for solving Example #2 are illustrated in Figure 12 below. Figure 12: The factors are (x^2+9) and (2x-3) Just like the last example, you can check to see if your final answer is correct by multiplying the factors together and seeing if the result equals the original cubic polynomial.Example #3: Factor $3y^3 + 18y^2 + y + 6$Finally, lets work through one more example where you have to factorize a cubic polynomial.Step One: Split the cubic polynomial into groups of two binomials.Again, the first step is to split the cubic polynomial down the middle into two binomials as shown in Figure 13 below. Figure 13: Factoring a cubic polynomial by grouping. As shown in Figure 13 above, splitting the polynomial down the middle leaves you with these two groups: $(3y^3 +18y^2)$ and (y+6)Remember that the whole point of splitting the cubic polynomial is to create two binomials that each have a GCF. But notice that the second binomial, (y+6), is not factorable because there is no GCF between +y and +6.But, as previously mentioned, this doesn't mean that you can not solve this problem further. In fact, the commutative property of addition allows you to swap the positions of the two middle terms ($18y^2$ and +y).This extra step of swapping the two middle terms is illustrated in Figure 14 below. Figure 14: Sometimes you have to swap the positions of the middle terms in order to factorize a cubic polynomial. After swapping the positions of the middle terms, you can now apply the 3-step method to factoring the equivalent polynomial: $3y^3 + y + 18y^2 + 6$ (this new cubic polynomial is equivalent to the original because the commutative property of addition allows you to rearrange the terms without changing the value of the expression).Now, you actually can split the new cubic polynomial into groups that can be factoring by dividing out a GCF: $(3y^3 + y)$ and $(18y^2 + 6)$ Figure 15: After swapping the positions of the middle terms, you can continue on with factoring the cubic polynomial. Step Two: Factor each binomial by pulling out a GCFAs shown in Figure 15 above, you can factor each group by pulling out a GCF as follows:Step Three: Identify the factors Finally, you can conclude that:Final Answer: The factors are (y+6) and $(3y^2 + 1)$The step-by-step process to solving this 3rd example are shown in Figure 16 below. Again, you can make sure that your final answer is correct by multiplying the factors together and verifying that their product is equivalent to the original cubic polynomial. Figure 16: How to factorize a cubic polynomial when you have to swap the middle terms. It is beneficial to understand how to factorize a cubic polynomial because the skill will allow you to simplify and understand the behavior of cubic functions as you continue onto higher levels of algebra and begin to explore topics like finding roots, analyzing graphs, and solving cubic equations.Factoring cubic functions can be challenging, but you can always use the following 3-step grouping method described in this guide to successfully factor a cubic polynomial (assuming that it is factorable in the first place):Step One: Split the cubic polynomial into groups of two binomials.Step Two: Factor each binomial by pulling out a GCFStep Three: Identify the factors Keep Learning: 1 Comment We can have two types of binomials cubed, a difference or a sum. The sum of cubes is an expression with the general form $latex {{a}^3}+{{b}^3}$ and a difference of cubes is an expression with the general form $latex {{a}^3}-{{b}^3}$. 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These entities provide the foundation for decomposing cubic binomials into their constituent factors, enabling efficient algebraic manipulations and problem-solving in mathematical contexts. Advanced Factoring Techniques: Unraveling the Mysteries of Polynomials Hey there, math enthusiasts! Are you ready to dive into the advanced realm of polynomial factoring? In this blog, we'll uncover some mind-boggling techniques that will make you feel like a factoring wizard. Let's get started! Method of Factoring Cubic Binomials: A Tricky Treat Cubic binomials, like x^3 + 2x^2 - 5x - 6, can be tricky to factor. But fear not! We've got a secret weapon: the method of factoring cubic binomials. This step-by-step approach involves finding two numbers that multiply to give the constant term (-6) and add to give the coefficient of the middle term (2). Once you've got those numbers (in this case, -3 and 2), you can factor the cubic binomial as (x - 3)(x^2 + 2x + 2). Factoring by Grouping: When Grouping Makes Sense Sometimes, polynomials can be factored by grouping terms with common factors. For instance, consider the polynomial x^4 - x^2 - 6x + 6. We can group the first two terms and the last two terms, giving us (x^4 - x^2) - (6x - 6). Now, we can factor out x^2 from the first group and 6 from the second group, yielding x^2(x^2 - 1) - 6(x - 1). Finally, we can factor the difference of squares in the first term and simplify the second term, resulting in x^2(x + 1)(x - 1) - 6(x - 1). And there you have it! The polynomial has been successfully factored. Advanced Polynomial Formulas Advanced Polynomial Formulas: Unraveling the Mysteries Hey math enthusiasts! Get ready to dive into the fascinating world of advanced polynomial formulas. These formulas are like magic tricks that can transform complex expressions into simpler forms. Let's explore the two most important ones: the sum of cubes formula and the difference of cubes formula. Sum of Cubes Formula Imagine a giant cube of sugar. If you have two of these sugar cubes, the sum of their volumes can be determined using the sum of cubes formula: (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 For example, let's say you have a sugar cube with a side length of 3 units and another with a side length of 2 units. The volume of the first cube is 3^3 = 27 cubic units, and the volume of the second cube is 2^3 = 8 cubic units. Using the formula, we can find the total volume of the two cubes: (3 + 2)^3 = 3^3 + 3(3^2)(2) + 3(3)(2^2) + 2^3 = 27 + 54 + 36 + 8 = 125 cubic units Ta-da! No need to build a tower of sugar cubes to get the answer. Difference of Cubes Formula Now, let's say you have two sugar cubes and you want to find the volume of the space between them. The difference of cubes formula comes to the rescue: (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 Using the same sugar cube example, the volume of the space between the cubes is: (3 - 2)^3 = 3^3 - 3(3^2)(2) + 3(3)(2^2) - 2^3 = 27 - 54 + 36 - 8 = *1 cubic unit* As you can see, the formula gives us the volume of the empty space between the cubes. It's like cutting a cake and seeing the empty space between the slices. These formulas are powerful tools that can make polynomial manipulation a breeze. So, embrace their magic and conquer the world of polynomials! Polynomial Division Hey there, math wizards! Let's dive into the world of polynomial division, where we'll learn a super cool trick called synthetic division. It's like a magic spell that makes polynomial division a piece of cake. Ready to be amazed? What's Synthetic Division? Imagine you have a difficult polynomial division problem like: (x^3 - 2x^2 + 5x - 6) ÷ (x - 2) Normally, this could give you a headache, but with synthetic division, it's as easy as making a cup of tea. Step-by-Step Magic 1. Line Up the Coefficients: Write the coefficients of the polynomial, starting with the highest power. Since we have an x^3 term, we need three spaces. Put a zero for the missing x^2 term. x^3 x^2 x const 1 -2 5 -6 2. Bring Down the Leading Coefficient: The number outside the parentheses (in this case, 2) goes in the corner. 2 | 1 -2 5 -6 3. Multiply and Add: Multiply the leading coefficient (1) by 2 and add the result to the next coefficient (-2). 2 | 1 -2 5 -6 ↓ ↓ 2 -4 4. Repeat for Next Coefficients: Repeat step 3 for the remaining coefficients. Multiply, add, and write the result below the line. 2 | 1 -2 5 -6 ↓ ↓ ↓ ↓ 2 -4 1 -2 5. Last Number Is the Remainder: The last number in the bottom row (-2) is the remainder. 6. Write the Answer: The answer to the polynomial division is written as a polynomial with the coefficients from the bottom row. (x^2 + 2x + 1) - 2/(x - 2) Easy peasy, right? Prime and Irreducible Polynomials: The Keys to Polynomial Mastery Yo, polynomial ninjas! Let's dive into the wild world of prime and irreducible polynomials, where factorization becomes an art form. So grab a cup of java and let's get started! Prime Polynomials: The Building Blocks Picture prime polynomials as the fundamental particles of polynomial factoring. They're like the atoms of polynomials, indivisible entities that form the basis for all other polynomial structures. Irreducible Polynomials: The Unbreakable Bonds Now meet their cousins, irreducible polynomials. These guys are like unbreakable fortresses, standing tall against factorization attempts. They can't be broken down into smaller polynomial pieces using any factoring techniques. The Criteria for Irreducibility But how do we know when a polynomial is irreducible? Well, the secret lies in three main criteria: It has no integer roots: No whole numbers make this polynomial equal to zero. It's not factorable over the integers: You can't find two integer polynomials that multiply to form it. It's not a product of other irreducible polynomials: No irreducible building blocks can be pieced together to make it. Examples of Prime and Irreducible Polynomials To wrap it up, let's check out some real-world examples: Prime Polynomials: Irreducible Polynomials: So, remember folks, prime and irreducible polynomials are the gatekeepers of polynomial factorization. They hold the key to simplifying complex expressions and unlocking the mysteries of the polynomial world. So, keep these concepts in mind, and your polynomial game will be off the charts! Alright, awesome work! You're now a pro at factoring cubic binomials. Remember, practice makes perfect, so don't be afraid to try out some more examples on your own. Thanks for hanging out with me today. If you have any more questions or need a refresher, feel free to drop by again later. I'm always happy to help. Catch you next time! Among the notable products, there is one that frequently appears in exercises, whether simple or more advanced: the one that allows you to expand binomial powers with exponent 3 and, at the same time, to factor certain types of quadrinomials.In this lesson, we study the cube of a binomial. We anticipate that it will haunt you throughout your school career! For this reason, it is good to remember the formula, as it allows you to limit the number of steps in algebraic calculations.Let's see the rule to apply for expansion and factorization, paying attention to the most common mistakes and discussing some worked examples.IndexRuleMinus signExpansion examplesFactorizationFactorization examplesHow to calculate the cube of a binomialLet's consider two monomials and that are not similar to each other, and consider the binomial given by their sum.The cube of a binomial is calculated as the cube of the first monomial, plus three times the product of the square of the first monomial and the second, plus three times the product of the first monomial and the square of the second, plus the cube of the second monomial. If there is a minus sign, we assign it to the respective monomial and perform the calculations according to the rule.To prove the formula, just express the power as a multiplication between polynomials and calculate it explicitly: Similarly, we can prove the rule for a generic binomial with a minus sign: Why is it useful and what is it for: since in algebraic calculations it often happens that you need to calculate powers of binomials with exponent 3, it is convenient to directly remember the cube of a binomial formula instead of doing the calculations every time. Special products serve precisely this purpose: to save calculations, effort, and time.A shortcut for the proofHere's a tip: if you don't remember the formula, you can quickly derive it by expanding the square of a binomial, which is easier to remember: We expand the square of a binomial: At this point, we multiply the two factors: Finally, we sum the like monomials: Cube of a binomial with differenceThe formula for the cube of a binomial with a minus sign is as follows: Instead of remembering the formula for expanding , we can limit ourselves to the one with the plus sign. In fact, we can use the formula for the cube of a binomial with a plus sign, assigning the minus sign to the second monomial. With this approach, we shift the minus sign to the respective monomial and apply the rule we already know. Note that the signs are found using the rule of signs for powers.In summary: we recommend not memorizing both formulas, as the first one is sufficient.Examples of binomial cubesLet's look at some examples. Starting with: We follow the rule and calculate the individual terms involved in the formula:- the cube of the first term is - the triple product of the square of the first term and the second is: - the triple product of the first term and the square of the second is: - the cube of the second term is: We will write: Example: binomial cube with fractional coefficientsLet's calculate the cube of a binomial with a fractional coefficient: Let's work out the various terms in the formula separately. Once we gain more experience, we will be able to calculate everything in one step, on a single line:- the cube of the first term is: - the triple product of the square of the first term and the second is: - the triple product of the first term and the square of the second is: - the cube of the second term is: We now have all the monomials needed to write the expansion: Example: binomial cube with monomials of degree greater than 1In this case, we have monomials of degree greater than 1. Here, we need to use the properties of powers, and in particular, the rule for powers of powers.- The cube of the first term is: - The triple product of the square of the first term and the second is: - The triple product of the first term and the square of the second is: - The cube of the second term is: We are ready to use the rule and compose the result: Common mistakes in expanding binomial cubesWhen working on exercises, most of the common mistakes are related to signs. The only way to avoid them is to practice a lot and always pay attention.Often, students forget to multiply by 3 in the products, or even completely forget about it. The only advice we can give is to remember that the expansion should result in a quadrinomial, i.e., it should have 4 terms.Factorization with the cube of a binomialTo factor a quadrinomial, you can try applying the expansion rule in reverse, that is, by reading the formula from right to left. We must check if the four terms correspond to those in the expansion of a binomial cube and, if so, express the quadrinomial as the cube of a binomial.Let's see how to proceed.We read it in reverse: We remind you that factoring a polynomial means expressing it as the product of other polynomials of positive degree and lower than the degree of the polynomial from which we started.The last equality we wrote allows us to express a particular type of quadrinomial as the product of three lower-degree polynomials, providing a factorization.In factorization exercises, when dealing with quadrinomials, we check if they correspond to a binomial cube, and if so, we factor them accordingly.We check if there are two cubes in the quadrinomial, and if so, we find the base monomials.We calculate the triple product of the square of the first base and the second base, the triple product of the first base and the square of the second base, and check if they match the other two terms of the quadrinomial.If so, the quadrinomial is indeed the expansion of a binomial cube, and we can rewrite it as the cube of the binomial formed by the two bases.Factorization examples with the cube of a binomialLet's try to factor the following polynomial: The presence of the two cubes and should immediately suggest a possible factorization using the cube of a binomial.We determine the bases of the two cubes and calculate the triple products to check if they match the remaining terms and of the original polynomial: The triple product of the square of the first base and the second is: The triple product of the first base and the square of the second is: These monomials match the other two terms of the original polynomial, so we have: Example: factoring and binomial cube factorizationAt a glance, we can see that the terms share a common factor , so we can rewrite it as a product between a monomial and a polynomial: What we applied is actually a factoring technique known as common factor extraction, which we will cover in detail in one of the following lessons.Let's focus on the polynomial in parentheses: Don't be fooled by the order of the terms: the two cubes and suggest that it could be the cube of a binomial. Let's calculate the bases: The triple product of the square of the first base and the second is: and this matches one of the remaining terms of the polynomial.The triple product of the first base and the square of the second is: and this matches the other remaining term. Therefore: and we can conclude that: The lesson ends here. In the next one, we will address the notable product of the difference of squares and, later, those for the sum of cubes and the difference of cubes.Allinllachu, see you soon guys! Fulvio Sbranchella (Agent Ω)Last edit: 21/10/2024 A binomial is an algebraic expression that has two terms in its simplified form. The word 'cube' of a number refers to a base raised to the power of 3. In this article, we will be studying the cube of a binomial which means a binomial being multiplied by itself 3 times. We will further be learning about the identities and formulas associated with the cube of a binomial. What is the Cube of a Binomial? The cube of a binomial is defined as the multiplication of a binomial 3 times to itself. We know that cube of any number 'y' is expressed as y × y × y or y3, known as a cube number. Therefore, given a binomial which is an algebraic expression consisting of 2 terms i.e., a + b, the cube of this binomial can be either expressed as (a + b) × (a + b) × (a + b) or (a + b)3. Cube of a Binomial Formula We will now be looking into the cube of a binomial formula. There are two formulas of the cube of a binomial depending on the sign between the terms. Those are given below. In the case of the cube of a binomial with an addition sign between the terms, we use the first formula which can be derived by multiplying the terms. (a + b)3 = (a + b) (a + b) (a + b) = (a2 + 2ab + b2) (a + b) = a3 + 3a2b + 3ab2 + b3 = a3 + 3ab(a + b) + b3 Thus, the cube of the sum of a binomial can be expressed as: (a + b)3 = a3 + 3ab(a + b) + b3. When it comes to the cube of a binomial with a subtraction sign in between, i.e a - b, we use the second formula - (a - b)3 = a3 - 3ab(a - b) - b3. (a - b)3 = (a - b) (a - b) (a - b) = (a2 - 2ab + b2) (a - b) = a3 - 3a2b + 3ab2 - b3 = a3 - 3ab(a - b) - b3 Thus, the cube of a binomial with a subtraction sign between the terms can be expressed as: (a - b)3 = a3 - 3ab(a - b) - b3. How to Solve Cube of a Binomial? Let's see the steps to solve the cube of the binomial (x + y). Step 1: First write the cube of the binomial in the form of multiplication (x + y)3 = (x + y)(x + y)(x + y). Step 2: Multiply the first two binomials and keep the third one as it is. (x + y)3 = (x + y)(x + y)(x + y) (x + y)3 = [x(x + y) + y(x + y)](x + y) (x + y)3 = [x2 + xy + xy +y2](x + y) (x + y)3 = [x2 + 2xy + y2](x + y) Step 3: Multiply the remaining binomial to the trinomial so obtained (x + y)3 = [x2 + 2xy + y2](x + y) (x + y)3 = x(x2 + 2xy + y2) + y(x2 + 2xy + y2) (x + y)3 = x3 + 2x2y + xy2 + x2y + 2xy2 + y3 (x + y)3 = x3 + 3x2y + 3xy2 + y3 (x + y)3 = x3 + y3 + 3x2y + 3xy2 (x + y)3 = x3 + y3 + 3xy(x + y) Related Articles Check these articles related to the concept of the cube of a binomial. Algebraic Expressions Cube Numbers Multiplication of Algebraic Expressions Example 1: Find the cube of the binomial (3x + 2y). Solution: We know that for a given binomial (a + b), the cube of the binomial (a + b)3 = a3 + 3ab(a + b) + b3. We will now be using this formula to evaluate (3x + 2y)3. Replacing a = 3x and b = 2y in the above formula we get, (3x + 2y)3 = (3x)3 + (2y)3 + 3(3x)(2y)(3x + 2y) = 27x3 + 8y3 + 18xy(3x + 2y) = 27x3 + 8y3 + 54x2y + 36xy2 Thus, the cube of the binomial (3x + 2y) is 27x3 + 8y3 + 54x2y + 36xy2. Example 2: If the value of (p + q) = 6 and pq = 8, find the value of p3 + q3. Solution: We know that, according to the cube of a binomial formula, Sum of cubes, a3 + b3 = (a + b)3 - 3ab(a + b) Replacing a = p and b = q, we get, p3 + q3 = (p + q)3 - 3pq(p + q) Given that, (p + q) = 6 and pq = 8. Substituting these in the above formula we get, p3 + q3 = 63 - 3 × 8 × 6 = 216 - 144 = 72 Thus, the value of p3 + q3 is 72. go to slidego to slide Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our certified experts Book a Free Trial Class FAQs on Cube of a Binomial A cube of a binomial is multiplying the binomial three times to itself. For example: (y + z)3 = (y + z) × (y + z) × (y + z). How to Expand Cube of a Binomial? Cube of a binomial can be expanded using the identities: (a + b)3 = a3 + 3ab(a + b) + b3 (a - b)3 = a3 - 3ab(a - b) - b3 What is the Product of the Cube of a Binomial? The product of the cube of a binomial is defined as multiplying the binomial 3 times with itself and expanding them to find the product as shown: (p + q)3 = (p + q) × (p + q) × (p + q) = p3 + 3p2q + 3pq2 + q3. What is the general form of the Cube of a Binomial? The general form of the cube of a binomial is given as: (x + y)3 = (x + y)(x + y)(x + y) = x3 + 3x2y + 3xy2 + y3. What are the Steps in Solving Cube of a Binomial? The steps to solve a cube of a binomial are given below: Step 1: First write the cube of the binomial in the form of multiplication (p + q)3 = (p + q) × (p + q) × (p + q). Step 2: Multiply the first two binomials and keep the third one as it is. Step 3: Multiply the remaining binomial to the trinomial so obtained. How do you Find the Cube of a Binomial? A cube of a binomial can be found by multiplying to itself three times. Or we can find the cube by using identities given below: (a + b)3 = a3 + 3ab(a + b) + b3 (a - b)3 = a3 - 3ab(a - b) - b3 In algebra class, the teacher would always discuss the topic of sum of two cubes and difference of two cubes side by side. The reason is that they are similar in structure. The key is to "memorize" or remember the patterns involved in the formulas. Case 1: The polynomial in the form [latex]{a^3} + {b^3}[/latex] is called the sum of two cubes because two cubic terms are being added together. Case 2: The polynomial in the form [latex]{a^3} – {b^3}[/latex] is called the difference of two cubes because two cubic terms are being subtracted. So here are the formulas that summarize how to factor the sum and difference of two cubes. Study them carefully. Case 1: Sum of Two Cubes Observations: For the "sum" case, the binomial factor on the right side of the equation has a middle sign that is positive. In addition to the "sum" case, the middle sign of the trinomial factor will always be opposite the middle sign of the given problem. Therefore, it is negative. Case 2: Difference of Two Cubes Observations: For the "difference" case, the binomial factor on the right side of the equation has a middle sign that is negative. In addition to the "difference" case, the middle sign of the trinomial factor will always be opposite the middle sign of the given problem. Therefore, it is positive. Examples of How to Factor Sum and Difference of Two Cubes Let's go over some examples and see how the rules are applied. Example 1: Factor [latex]{x^3} + 27[/latex]. Currently, the problem is not written in the form that we want. Each term must be written as a cube, that is, an expression raised to a power of [latex]3[/latex]. The term with variable [latex]x[/latex] is okay but the [latex]27[/latex] should be taken care of. Obviously, we know that [latex]27 = \left(3 \right)\left(3 \right)\left(3 \right) = {3^3}[/latex]. Rewrite the original problem as sum of two cubes, and then simplify. Since this is the "sum" case, the binomial factor and trinomial factor will have positive and negative middle signs, respectively. Example 2: Factor [latex]{y^3} – 8[/latex]. This is a case of difference of two cubes since the number [latex]8[/latex] can be written as a cube of a number, where [latex]8 = \left(2 \right)\left(2 \right)\left(2 \right) = {2^3}[/latex]. Apply the rule for difference of two cubes, and simplify. Since this is the "difference" case, the binomial factor and trinomial factor will have negative and positive middle signs, respectively. Example 3: Factor [latex]27{x^3} + 64{y^3}[/latex]. The first step as always is to express each term as cubes. We know that [latex]27 = {3^3}[/latex] and [latex]64 = {4^3}[/latex]. Rewrite the problem as sum of two cubic terms and apply the rule, so we get Example 4: Factor [latex]125{x^3} – 27[/latex]. Since [latex]125 = \left(5 \right)\left(5 \right)\left(5 \right) = {5^3}[/latex] and [latex]27 = \left(3 \right)\left(3 \right)\left(3 \right) = {3^3}[/latex], this is clearly a problem on difference of two cubes. Here's the solution. Example 5: Factor [latex]1 – 216{x^3}{y^3}[/latex]. At first, this problem may look "difficult". However, if you stick to what we know already about sum and difference of two cubes we should be able to recognize that this problem is rather easy. The good thing is that the variables are cubes so they are fine. Now for the number, it is easy to see that that [latex]1 = \left(1 \right)\left(1 \right)\left(1 \right) = {1^3}[/latex] while [latex]216 = \left(6 \right)\left(6 \right)\left(6 \right) = {6^3}[/latex]. This is really a case of difference of two cubes. Example 6: Factor [latex]8{x^6}{y^{12}} + 27[/latex]. This problem is a bit different. The coefficients are definitely cubes because [latex]8 = {2^3}[/latex] and [latex]27 = {3^3}[/latex]. Now, how do we express the term with variables as a cube? Well, simply factor out [latex]3[/latex] from the existing exponents of "[latex]x[/latex]" and "[latex]y[/latex]". Use the law of exponent known as Power to a Power Rule to justify this step. We take out [latex]3[/latex] because to be a cube implies that any expression must have an outer exponent of [latex]3[/latex]. Example 7: Factor [latex]3xy – 24{x^4}y[/latex]. Sometimes the problem may not appear to be factorable by either sum or difference of two cubes. If you see something like this, try to take out common factors. For the numbers, the greatest common factor is [latex]3[/latex] and for the variables, the greatest common factor is "[latex]xy[/latex]". Therefore the overall common factor would be their product which is [latex]\left(3 \right)\left({xy} \right) = 3xy[/latex]. After factoring it out, you'll see that we have an easy problem on the difference of two cubes. You might also be interested in: Tags: Intermediate Algebra, Lessons