By Shailendra Singh|Updated on 27 Dec 2024, 14:00 ISTObserving the free oscillations of a real physical system, on the other hand, reveals that the oscillator's energy gradually decreases with time, and the oscillator eventually comes to rest. For example, the amplitude of an oscillating pendulum in the air decreases over time and eventually stops. With the passage of time, the vibrations of a tuning fork fade away. This occurs because friction (or damping) is always present in physical systems. Friction is the resistance to motion. The presence of resistance to motion implies that the system is subjected to a frictional or damping force. The damping force acts in opposition to the motion, doing negative work on the system and causing energy to be dissipated. Fill out the form for expert academic guidanceWhen a body moves through a medium such as air, water, or ice, its energy is dissipated due to friction and appears as heat in the body, the surrounding medium, or both. Another mechanism by which an oscillator loses energy exists. An oscillator's energy can be reduced not only by friction in the system but also by radiation. The oscillating body causes periodic motion in the particles of the medium in which it oscillates, resulting in waves.Save 10% Now! Enter Code: AITS10OFFUnlock the full solution & master the conceptGet a detailed solution and exclusive access to our masterclass to ensure you never miss a conceptBoost Your Preparation With Our All India Test Series for JEE/NEET 2025A tuning fork, for example, generates sound waves in the medium, causing its energy to decrease. All sounding bodies are subject to dissipative forces; otherwise, the body would lose no energy and therefore emit no sound energy. As a result of the radiation of mechanical oscillatory systems, sound waves are produced. We will discover later that electromagnetic waves are created by radiations from oscillating electric and magnetic fields.The amplitude of oscillations gradually decreases with time as a result of radiation by an oscillating system and friction in the system. The reduction in amplitude (or energy) of an oscillator is referred to as damping, and the oscillation is referred to as being damped.Ready to Test Your Skills?Check Your Performance Today with our Free Mock Tests used by Toppers!An oscillator is defined as anything with a rhythmic periodic response. A damped oscillation is one that gradually fades away with time. A swinging pendulum, a weight on a spring, and a resistor-inductor-capacitor (RLC) circuit are all examples. Assume we have an RLC circuit with a resistor, inductor, and capacitor in series. The capacitor will discharge into a series resistor and inductor when the switch is closed at time t=0. Now, the voltages and currents in this circuit can be calculated usingand V = starting voltageC stands for capacitance (farads)R stands for resistance (ohms)L stands for inductance (henrys)e = natural log base (2.71828...) Start Your JEE/NEET Prep at Just ₹1999 / month - Limited Offer! Check Now!The current for a damped sine wave is given by the equation above. It is a maximum amplitude sine wave (V/BL) multiplied by an exponential decay damping factor. The time variation that results is an oscillation bounded by a decaying envelopeThe motion of a damped harmonic oscillator is governed by the second-order differential equation:create your own testYOUR TOPIC, YOUR DIFFICULTY, YOUR PACE$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$m: Mass of the oscillating bodyb: Damping coefficient (measure of resistive force)k: Spring constant (restoring force constant)x: Displacement from equilibriumdx/dt: Velocity$d^2x/dt^2$: AccelerationThe solution depends on the damping ratio (ζ), defined as:The system oscillates with exponentially decreasing amplitude:$x(t) = A e^{-\zeta \omega_0 t} \cos(\omega_d t + \phi)$A: Initial amplitude$\omega_0 = \sqrt{k/m}$: Natural angular frequency$\omega_d = \omega_0 \sqrt{1 - \zeta^2}$: Damped angular frequencyφ: Phase constantThe system returns to equilibrium as quickly as possible without oscillating:$x(t) = (C_1 + C_2 t)e^{-\omega_0 t}$The system does not oscillate and slowly returns to equilibrium:$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$r_1$, $r_2$: Roots of the characteristic equation$r_1, r_2 < 0$The total energy of the system decreases exponentially over time:$E(t) = E_0 e^{-2\zeta \omega_0 t}$Key FeaturesThe frequency of damped oscillation ($\omega_d$) is lower than the natural frequency ($\omega_0$) of the undamped system.The degree of damping determines whether the system oscillates ($\zeta < 1$), returns critically ($\zeta = 1$), or overdamps ($\zeta > 1$).Damped oscillations are common in real-world systems, such as car suspension systems, clocks, and building vibrations during earthquakes.FAQsDamped oscillation refers to an oscillation that fades over time. The amplitude of the oscillation decreases over time due to damping. The system's amplitude is reduced as a result of energy lost in overcoming external forces such as friction, air resistance, and other resistive forces.Damped harmonic oscillators are vibrating systems whose amplitude decreases over time. AnswerVerified Hint: In simple words, damped oscillations is the type of oscillations that diminishes in amplitude with time. Here, in this question, we will first understand the concept involved in oscillations and then, explain damped oscillations with an appropriate example. Complete step by step solution:The regular change in location and/or magnitude around a central point or a mean position is known as oscillation. In general, oscillations are expressed in Hertz. Example: Simple pendulum, tuning forks, guitar strings are some of the examples of undertaking oscillatory motion.Types of oscillations:1. Damped Oscillation2. Forced Oscillation3. Free OscillationDamped oscillation is an oscillation that diminishes with time. The amplitude of oscillations decreases with time due to damping. The oscillation is given resistance by the damping. The loss of energy from the system in resisting external forces such as friction, air resistance, and other resistive factors causes a reduction in amplitude. As a result, as the amplitude of the system decreases, so does its energy.Example: The motion of the oscillating pendulum kept inside an oil-filled tank.Image: Simple PendulumHere, a simple pendulum of mass "$m$" is in the damped oscillation motion such that the length of the string attached is "$L$". The restoring force in the damped oscillations will always remain the same (constant) with respect to time and displacement. Here, the damping force always acts opposite to the motion of the bob (pendulum) which decreases the oscillation. When the pendulum is moving from right-hand side to left-hand side then, the damping force will be the $mg\cos\theta$ while the pendulum is moving from left-hand side to right-hand side then, the damping force will be the $mg\sin\theta$.Note: Candidates get confused with the term's simple harmonic motion and oscillatory motion. These two terms are completely different in the sense that the restoring force in the simple harmonic motion is directly proportional to the displacement whereas in oscillatory motion (damped), the restoring force is constant. Explain damped oscillation. Give an example.The oscillations in which the amplitude decreases gradually with the passage of time are called damped oscillations. Example: The oscillations of a pendulum or pendulum oscillating inside an oil-filled container. Electromagnetic oscillations in a tank circuit. Oscillations in a dead beat and ballistic galvanometers. shaalaa.com  Is there an error in this question or solution?Page 2Define forced oscillation. Give an example.The body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations. Example: Sound boards of stringed instruments.shaalaa.com  Is there an error in this question or solution?Page 3What is meant by maintained oscillation? Give an example.While playing in swing, the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations. Example: The vibration of a tuning fork getting energy from a battery or from an external power supply.shaalaa.com  Is there an error in this question or solution? Damped Oscillation means the oscillating system experiences a damping force, causing its energy to decrease gradually. The level of damping affects the frequency and period of the oscillations, with very large damping causing the system to slowly move toward equilibrium without oscillating. In this article, we will look into damped oscillation, damped oscillator, damping force, general equation derivation, application and type of damped oscillation, etc. What is Damped Oscillation? Damped oscillation refers to the condition in which the amplitude of an oscillating system gradually decreases over time due to the dissipation of energy by non-conservative forces, such as friction or air resistance. This phenomenon is observed in various systems, such as mass oscillating on a spring or shock absorbers in a car. Damped oscillations are classified as underdamped, critically damped, and overdamped based on the damping constant. The damping force causes the amplitude of the oscillations to decrease gradually, leading to a loss of energy from the system. Definition of Damped OscillationDamped oscillation refers to an oscillatory motion in which the amplitude of the oscillation gradually decreases over time. This decrease in amplitude is due to the dissipation of energy from the system, often due to friction or other resistive forces. Damped OscillatorA simple harmonic oscillator whose amplitude of vibrations always reduces with time and, finally, the system comes to rest due to the unavoidable presence of damping force is called a Damped oscillator. Damping Force Damping force is the external force present in the surroundings, which acts against the oscillatory motion of the system. The magnitude of the damping force determines the way in which the motion proceeds. The damping force is always directly proportional to the velocity of the oscillatory system. Damped Oscillation Differential EquationThe equation of motion for a damped harmonic oscillation is a second-order ordinary differential equation. It can be expressed as: $md^2x/dt^2 + 2\mu.dx/dt + \omega 02x(t) = 0$ Where: m is the mass of the object,x(t) is the position of the object as a function of time,2μ is the damping coefficient,ω0 is the natural frequency of the oscillatorDamped Harmonic OscillatorConsider a body of mass' m' is set to the oscillatory motion in air with velocity v. Let 'k" be the restoring force constant of the system. For ideal SHM (neglecting any opposition/ damping), the differential equation of motion is written using Newton's 2nd law m.a = Ftotal $m.d^2x/dt^2 = -kx$......(1) In real situation, there is always some resistance offered to moving body such that. The damping force is directly proportional to the velocity of the oscillatory body. Damping force = -bv ..................(2) Where b = damping constant Considering damping force in the equation 1, we get $m.d^2x/dt^2 = -kx - bv$.....(3) Using equation 3 and rearranging, we get Dividing both sides with 'm' and rearranging, we get $d^2x/dt^2 = -kx/m - bv/m$ $d^2x/dt^2 = -(k/m)x - (b/m)v$ $d^2x/dt^2 = -(k/m)x - (b/m)dx/dt$ $d^2x/dt^2 + (k/m)x + (b/m)dx/dt = 0$......(4) Let $\sqrt{(k/m)} = \omega 0$ And $b/m = 2\mu$ $d^2x/dt^2 + \omega 02x + 2\mu.dx/dt = 0$....(5) ω0: natural frequency (angular) of the system. μ: damping factor (damping force per unit mass at any instant when vibrating body moving with unit velocity) Equation 5 is called the differential equation of motion for a damped oscillatory system, which describes the motion of any damped oscillatory motion. Using differential operator D = d/dt, equation 5 is reduced to the following. $D^2x + \omega 0x + 2\mu xD = 0$ $(D^2 + \omega 0 + 2\mu D)x = 0$.....(6) Here, x represents displacement at any instant of time; hence $x \neq 0$ $(D^2 + \omega 02 + 2\mu D) = 0$......(7) Solving equation 7 as a quadratic equation, we get $D = \{-2\mu \pm \sqrt{[(2\mu)^2 - 4\omega 0^2]}\}/2$ $D = -\mu \pm \sqrt{\mu^2 - \omega 0^2}$.....(8) As shown in equation 8, D has two possible values of the square root $\alpha = -\mu + \sqrt{\mu^2 - \omega 0^2}$ and $\beta = -\mu - \sqrt{\mu^2 - \omega 0^2}$ D = d/dt $Dx = dx/dt = dx/dt = \alpha x$....(8a) and $dx/dt = \beta x$....(8b) Consider equation 8a; rearranging and integrating, we get $\int dx/dt = \alpha x$ and $\int dx/dt = \beta x$ $\int dx/x = \int \alpha dt = Ae\alpha t$....(9a) $\int dx/x = \int \beta dt = Be\beta t$....(9b) Equation 9a and 9b are both possible solutions of the differential equation (equation 5); hence, the general solution is given as $x(t) = Ae\alpha t + Be\beta t$ $x(t) = \{Ae[-\mu + \sqrt{\mu^2 - \omega 0^2}]t + Be[-\mu - \sqrt{\mu^2 - \omega 0^2}]t\}$ $x(t) = e\text{-}\mu t\{Ae[\sqrt{\mu^2 - \omega 0^2}]t + Be[-\sqrt{\mu^2 - \omega 0^2}]t\}$...(10) Equation 10 is the general solution of the differential equation of motion (equation 5) for the damped oscillator. It helps us to determine the displacement' x' of the oscillatory system at any instant in time. In this equation, A and B are the constants that depend on the oscillatory system. The term e-μt represents the exponential decay of the displacement (amplitude) with respect to time. Damped Oscillation Equation The general equation of differential equation can be represented as follows: $x(t) = e\text{-}\mu t\{Ae[\sqrt{\mu^2 - \omega 0^2}]t + Be[-\sqrt{\mu^2 - \omega 0^2}]t\}$ Damped Oscillation FormulaThe equation gives the formula for the damped oscillation of a harmonic oscillator: $x(t) = x0e - \mu\omega 0t \cos(\omega dt + \phi)$ Where, x(t) is the displacement at time x0 is the initial displacement,μ is the damping ratio,ω0 is the undamped angular frequency,ωd is the damped angular frequency, andϕ is the phase angle.Damped Harmonic Oscillator CasesIn a damped harmonic oscillator, three cases are distinguished based on the damping level: Large Damping: In systems with very large damping, oscillations do not occur; instead, the system slowly moves towards equilibrium. The displacement of the oscillator moves more slowly towards equilibrium than critically damped systems.Critical Damping: Critical damping occurs when the damping constant equals the square root of 4 times the mass multiplied by the spring constant. Systems under critical damping return to equilibrium as quickly as possible, like shock absorbers in cars, without overshooting.Small Damping: In underdamped systems, oscillations occur while the amplitude decreases exponentially until the system comes to rest. These systems oscillate through the equilibrium position and eventually approach zero amplitude.Damped Oscillations Having One Degree of Freedom A damped oscillation of a system with one degree of freedom refers to the behavior of a simple system with one moving part subject to a linear viscous damping force. The system is assumed to have a small velocity, and the damping force is proportional to the velocity. The system's motion can be described by a differential equation known as the damped harmonic oscillator equation, which can be solved to find the displacement and velocity of the system as a function of time. $d^2x/dt^2 = -(k/m)x - (b/m)dx/dt$ This equation is for small displacements and velocities. The equation can be rewritten to: $md^2x/dt^2 + \omega 02x(t) + 2\mu.dx/dt = 0$ The system's behavior depends on the character of the roots of the equation, which may be real or complex. The damping present in the system can be characterized by the quantity gamma, which has the dimension of frequency, and the constant ω0 represents the natural angular frequency of the system in the absence of damping. The system's behavior can be heavily, weakly, or critically damped, depending on gamma values and ω0. Types of Damped Oscillator Damped oscillators are classified into three main types based on the damping ratio: overdamped, critically damped, and underdamped. Overdamped OscillatorIn this type, the system returns to equilibrium without oscillating. The motion decays more slowly toward equilibrium than in a critically damped system. The oscillatory system, where the damping force experienced by the system from the surroundings is larger than the restoring force of the system such that ($\mu > \omega 0$) is called Overdamped oscillation. The equation of displacement for overdamped oscillation is $x(t) = e\text{-}\mu t\{Ae[\sqrt{\mu^2 - \omega 0^2}]t + Be\text{-}[\sqrt{\mu^2 - \omega 0^2}]t\}$ Overdamped motion is non-oscillatory. In this case, the amplitude decreases exponentially, reaching equilibrium very slowly. (In an ideal case, it takes infinite time to reach equilibrium.) Critically Damped OscillatorHere, the system returns to equilibrium as quickly as possible without oscillating. Shock absorbers often exemplify this type in cars, where oscillations decay rapidly. The oscillatory system, where the damping force experienced by the system from the surroundings is well balanced by the restoring force of the system such that ($\mu^2 = \omega 0$) is called a critically damped oscillation. The time rate equation of displacement for critically damped oscillation is $X(t) = e\text{-}\mu t\ (A + Bt)$ Critically damped motion is non-oscillatory. In this case, the amplitude decreases exponentially, reaching equilibrium much faster than over-damped conditions. Underdamped OscillatorThis type oscillates at a frequency slightly different than the undamped case and gradually returns to equilibrium. An example is a weight on a spring with some damping, where the motion slowly comes to rest. The oscillatory system, where the damping force experienced by the system from surrounding is less than the restoring force of the system such that (μ