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Rolling resistance is the resistive force applied to the wheel when its rolling. The magnitude of rolling resistance depends on a series of factors, which most of the time act together and can not be broken down clearly. Most common causes of rolling resistance applied to a wheel in motion are: tire deformation friction between tire and road surface wheel friction with surrounding air friction in the wheel hub and bearings road surface deformation In the following discussion we are going to focus on the rolling resistance caused by the tire and road. The wheel hub and bearings efficiency is not part of this lecture since they can be consider as part of the overall driveline losses (efficiency). Image: Tire pressure distribution stationary vs. rolling In the image above: N [N] normal reaction force, W [N] weight force acting on the wheel and [rad/s] angular speed of the wheel. When the tire is stationary, the distribution of the normal force N [N] in the contact patch is symmetric with respect with the vertical axis of the wheel. However, when the wheel is in motion (rotating), the normal force distribution is not anymore symmetric but concentrates in the front of the wheel, towards the direction of rotation. This makes the resultant force N [N] to create a resistive rolling torque, which will try to slow down the wheel. Go back Formula Image: Rolling resistance wheel forces diagram Lets assume we have a vehicle which has front-wheel drive (FWD). The rear wheels are going to be pulled along, through the chassis. Therefore there will be a pulling force Fp [N] acting in the wheel hub, which will force the wheel to rotate. As explained previously, due to asymmetric force distribution in the contact patch, there is going to be a normal force N [N] acting on the tire, at the distance a [m] from the vertical axis of the tire. The force N [N] is the vertical component of a resultant force going through the tires centre of rotation. The horizontal component of that resultant force, acting in the contact patch, which is trying to slow down the tire is exactly the rolling resistance force Frr [N]. Since the wheel is in equilibrium, the sum of forces on the x-axis, the sum of forces on the y-axis and the sum of torques acting around the centre of the wheels are all zero. x-axis forces equilibrium y-axis forces equilibrium Replacing N from (6) in (8) and solving for Frr [N] gives: The ratio between distance a [m] and wheel radius rw [m] is the rolling resistance coefficient f [-]. Replacing (10) in equation (9) gives the general formula of the rolling resistance force for flat (no gradient) roads. where m [kg] is the vehicle mass and g = 9.81 m/s2 is the gravitational acceleration. If we calculate the total rolling resistance force of the vehicle, then the entire vehicle mass is used in the equation. If we calculate the rolling resistance of only one wheel, then the vehicle mass is divided by four (assuming equal weight distribution between wheels). If the vehicle is rolling on a road with the gradient [], then the formula for rolling resistance becomes: Frr = f W cos() = f m g cos() Go back Rolling resistance coefficient The rolling resistance coefficient of a tire depends on tire construction, wheel materials, air pressure, vehicle speed, and road conditions. In general, for low vehicle speeds, the value of rolling resistance coefficient is constant. The rolling resistance coefficient slightly increases with the increase of the traveling speed of the vehicle.According to [7], under normal operating conditions, with the vehicle speed below 200 kph, the rolling resistance coefficient is between 0.01 0.02. There are several sources of automotive literature, in which you can find the value of the rolling resistance coefficient, function on several parameters. For example in [6], you can find the value of the rolling resistance coefficient function of the road type. Road surface Rolling resistance coefficient Pneumatic car tires on large set pavement 0.013 small set pavement 0.013 concrete, asphalt 0.011 rolled gravel 0.02 tarmacadam 0.025 unpaved road 0.05 field 0.1 0.35 Pneumatic truck tires on concrete, asphalt 0.006 0.01 Strake wheels in field 0.14 0.24 Track-type tractor in field 0.07 0.12 Wheel on rail 0.001 0.002 Table: Coefficients of rolling resistance Source: [6] From [1] we can also extract a table of rolling resistance coefficients function of road surface. Road surface Rolling resistance coefficient new, firm asphalt; concrete; small pavement; cobblestone pavement 0.005 0.015 rolled, firm gravel; wear down, washboard asphalt 0.02 0.03 tarred, wear down, washboard gravel 0.03 0.04 very well dirt roads 0.04 0.05 dirt roads 0.05 0.15 sand 0.15 0.35 Table: Coefficients of rolling resistance Source: [1] Additional values for the rolling resistance coefficient can be found in [8] for different pavement types at low vehicle speeds. Pavement type Rolling resistance coefficient Good asphalt or concrete pavement 0.01 0.018 General asphalt or concrete pavement 0.018 0.02 Gravel road 0.02 0.025 Good gravel road 0.025 0.030 Pebble potholes pavement 0.035 0.050 Pressed dirt road (dry) 0.025 0.035 Pressed dirt road (rainy) 0.050 0.150 Muddy dirt road 0.100 0.250 Dry sand 0.100 0.300 Wet sand 0.060 0.150 Icy roads 0.015 0.030 Compacted ski track 0.030 0.050 Table: Rolling resistance coefficient of a vehicle at low speed on a given road Source: [8] In reality, the rolling resistance coefficient depends on several factors, like: tire construction road type slip between tire and road inflating pressure wheel load In the image below you can see the influence of tire pressure on the rolling resistance coefficient. As the air pressure increases, the rolling resistance coefficient decreases. This happens because an increased internal pressure leads to a further stiffening of the tire, which means that the tire deflection decreases at steady load. This leads to a decrease of the tire flexing energy and due to the smaller contact patch, to a decline of the frictional component of resistance [1]. Image: Rolling resistance coefficient function of wheel load and tire pressureCredit: [1]In the next image you can see the influence of vehicle speed on the rolling resistance. The increase of the rolling resistance coefficient, which rises with the vehicle speed is due to the superimposed effect of the tire deformation wave on the flexion resistance. This effect increases with velocity [1]. Image: Rolling resistance coefficient function of driving speedCredit: [1] Go back Models for rolling resistance coefficient For calculation/simulation purposes, you can use either a constant value of the rolling resistant coefficient or a speed dependant one. In the automotive literature, there are a number of formulas for the rolling resistance coefficient, mainly function of vehicle speed and tire pressure. Rolling resistance coefficient function of vehicle speed In [9] there are several proposed formulas for the rolling resistance coefficient. \(f (V)= f_{0} + f_{01} \cdot V + f_{02} \cdot V^{2} \tag{13}\) where V [kph] is vehicle speed and f0, f01 and f02 are coefficients function of tire construction. For example, for a radial tire, the coefficients can be set as: f0 = 1 10-2 f01 = 5 10-7 f02 = 2 10-7 If we plot f [-] function of V [kph], we get the following curve: Image: Rolling resistance coefficient (1) The coefficients can be adjusted in order to obtain the desired behaviour of the rolling resistance coefficient. For a road made from good asphalt or concrete, a rolling coefficient which varies between 0.01 at 0 kph and 0.025 at 250 kph is good enough for simulations. Go back Models for rolling resistance coefficient Once we have the rolling resistance coefficient function of vehicle speed, we can apply equation (12) and get the rolling resistance force. For example, if we consider that the total mass of the vehicle is 2000 kg and that the weight is evenly distributed on all four wheels, we get a weight on a wheel of: Frr = (2000/4) 9.81 f(V) Applying equation (12) and the rolling resistance coefficient defined by (13), we can plot the rolling resistance force function of vehicle speed. Image: Rolling resistance force function of speed In the SAE standard J2452, the rolling resistance is defined function of vehicle speed, tire inflation pressure and load applied on the wheel. Frr = Z P (a + b V + c V2) where: Frr [N / lbs] rolling resistance force P [kPa / psi] tire inflation pressure Z [N / lbs] tire load V [kph / mph] vehicle speed , , a, b, c [-] tire dependant coefficients As an example, from [3] we can extract the coefficients for the T40 of size 205/75R15 97S: = 1.03399904 = -0.41081927 a = 0.05933157 b = 9.85526e-05 c = 3.72314e-07 For the same tire load of 4905 N and different inflation pressures of 150, 250 and 350 kPa, we can plot the rolling resistance using equation (15): Image: Rolling resistance force function of speed pressure and load Go back Example For a vehicle of mass 1800 kg, travelling on a road with the slope angle of 10 and with a fixed rolling resistance coefficient of 0.012, calculate the total rolling resistance force. Applying equation (12), we get: Frr = 1800 9.81 0.012 cos(10 / 180) = 209 N Go back Calculator Go back References [1] Henning Wallentowitz, Longitudinal Dynamics of Vehicles Lecture, IKA RWTH, Aachen, 2004. [2] Lars Eriksson, Lars Nielsen, Modeling and Control of Engines and Drivelines, Wiley, 2014. [3] Ben Wen, Gregory Rogerson, and Alan Hartke, Correlation Analysis of Rolling Resistance Test Results from SAE J1269 and J2452, SAE International, 2014. [4] Green Seals Report, Low Rolling Resistance Tires, March, 2003. [5] Transportation Research Board, Tires and Passenger Vehicle Fuel Economy, Special Report 286, 2006. [6] Automotive Handbook, 9th Edition, Bosch, 2014. [7] Georg Rill, Vehicle Dynamics Lecture Notes, University of Applied Sciences, Regensburg, 2005. [8] Wuwei Chen et al, Integrated Vehicle Dynamics and Control, Wiley, 2016. [9] M. Untaru et al, Dinamica Autovehiculelor pe Roti, Editura Didactica si Pedagogica, Bucuresti, 1981. Dear user, Our website provides free and high quality content by displaying ads to our visitors. Please support us by disabling your Ad blocker for our site. Thank you! NAFEMS Americas and Digital Engineering (DE) teamed up (once again) to present CAASE, the (now Virtual) Conference on Advancing Analysis & Simulation in Engineering, on June 16-18, 2020!CAASE20 brought together the leading visionaries, developers, and practitioners of CAE-related technologies in an open forum, unlike any other, to share experiences, discover common themes, and explore future issues, including:-What is the future for engineering analysis and simulation?-Where will it lead us in the next decade?-How can designers and engineers realize its full potential?What are the business, technological, and human enablers that will take past successful developments to new levels in the next ten years?Resource AbstractVehicle Energy Management (VEM) simulations play key role in prediction of System level energy performance and energy contribution of each subsystem during any event. Tire-road interaction plays crucial role in driving the actual power consumption of vehicle. This is so, due to dynamic behavior of tire material when each tread gets loaded (comes into contact with ground) and unloaded (loses contact with ground). The rolling resistance between tire-ground also directly affects fuel consumption of vehicle. Slight change in rolling resistance significantly impacts power consumption. Also, this makes virtual simulation deviate a lot from physical world. Thus, calculation of rolling resistance is very critical for good correlation. Rolling resistance depends on various factors such as load, vehicle velocity and ambient conditions. One of the conventional approaches to estimate the rolling resistance is by trial and error method. This paper proposes a scientific approach for calculation of rolling resistance coefficient for various off-road events. It is clear that the rate of tire tread being loaded and unloaded depends on forward vehicle velocity. Also, vertical weight on the tire directly impacts on the shape and size of contact patch area. Thus, rolling resistance being offered at high load is more. A new methodology has been developed to calculate rolling resistance coefficient for various duty cycles in which vehicle forward velocity and vertical load are varying. This methodology has been virtually verified for field and transport events of an off-road vehicle and excellent correlation (around 90%) has been achieved with test data. Standardization of this process is a key milestone towards predicting performance of upcoming off-road machines for various terrains and vehicle system level parameters. This study is a step towards modeling real time vehicle dynamic phenomenon accurately and making sound design decisions based upon results achieved. Further scope of this study is to optimize hydraulic subsystems based upon subsystem level performance. This will lead to performance improvement at vehicle level significantly. With this method, huge cost involved in testing and efforts will be minimized with added advantage of reusability and modular architecture. Evaluation of various technological advancements at subsystem level can be appropriately compared with available options using vehicle energy management thought process. Also, optimization objectives such as better gradeability, improved traction, reduced fuel consumption and emissions can be achieved with this methodology. The force that resists the motion of a body rolling on a surface is called the rolling resistance or the rolling friction.The rolling resistance can be expressed by the generic equationFr = c W (1)where Fr = rolling resistance or rolling friction (N, lbf)c = rolling resistance coefficient - dimensionless (coefficient of rolling friction - CRF)W = m ag = normal force - or weight - of the body (N, lbf)m = mass of body (kg, lb)ag = acceleration of gravity (9.81 m/s2, 32.174 ft/s2)Note that the rolling resistance coefficient - c - is influenced by different variables like wheel design, rolling surface, wheel dimensions and more. Both rolling body and base are subject to deformations during rolling and large bodies rolls more easily than small bodies. Harder roller and base surfaces results in lower resistance coefficients. The rolling resistance coefficient for a car tire on asphalt is approximately 0.2 and the rolling resistance for a ball bearing is approximately 0.002.The rolling resistance can alternatively be expressed asFr = cl W / r (2)where cl = rolling resistance coefficient - dimension length (coefficient of rolling friction) (mm, in)r = radius of wheel (mm, in) Rolling Friction CoefficientsSome typical rolling coefficients:Rolling Friction CoefficientsRolling Resistance Coefficientccl(mm)0.001 - 0.0020.5railroad steel wheels on steel rails, ball bearings0.001bicycle tire on wooden track0.002 - 0.005low resistance tubeless tires0.002bicycle tire on concrete0.004bicycle tire on asphalt road0.005dirty tram rails0.006 - 0.01truck tire on asphalt0.008bicycle tire on rough paved road0.01 - 0.015ordinary car tires on concrete, new asphalt, cobbles small new0.02car tires on tar or asphalt0.02car tires on gravel - rolled new0.03car tires on cobbles - large worn0.04 - 0.08car tire on solid sand, gravel loose worn, soil medium hard0.2 - 0.4car tire on loose sandRolling Coefficients CarsThe rolling coefficients for air filled tires on dry roads can be estimatedc = 0.005 + (1 / p) (0.01 + 0.0095 (v / 100)2) (3)where c = rolling coefficientp = tire pressure (bar)v = velocity (km/h) Example - Wheel Pressure & Rolling Resistance CoefficientThe standard wheel pressure in a Tesla Model 3 is 2.9 bar (42 psi). The rolling friction coefficient at 90 km/h (56 mph) can be calculated from (3) asc = 0.005 + (1 / (2.9 bar)) (0.01 + 0.0095 ((90 km/h) / 100)2) = 0.011Increasing the pressure to 3.5 bar reduces the rolling resistance coefficient toc = 0.005 + (1 / (3.5 bar)) (0.01 + 0.0095 ((90 km/h) / 100)2) = 0.010- or((0.011 - 0.10) / 0.011) 100% = 9%1 bar = 105 Pa = 14.5 psi1 km/h = 0.6214 mphExample - The Rolling Resistance of a Car on AsphaltThe rolling resistance for all four wheels in a car with total weight 1500 kg on asphalt with rolling friction coefficient 0.03 can be estimated with the generic equation 1 asFr = 0.03 (1500 kg) (9.81 m/s2) = 441 N = 0.44 kNcompare car rolling resistance with car air resistance (drag)The rolling resistance for one wheel can be calculated asFr = 0.03 (1500 kg / 4) (9.81 m/s2) = 110 N = 0.11 kN Change in velocity vs. time used. Required forces to move bodies up inclined planes. Power, torque, efficiency and wheel force acting on a car. Adhesion and tractive force between car wheel and surface. Car acceleration calculator. Calculate fuel consumption in liter per km - consumption chart and calculator. Forces due to circular motion and centripetal / centrifugal acceleration. Forces and torque activated with disk brakes. The drag coefficient quantifies the drag or resistance of an object in a fluid environment. Friction and efficiency in bearings and roller chains. EV Charging - AC vs. DC, single phase vs. three phase and power vs. voltage and amps. Linear and angular (rotation) acceleration, velocity, speed and distance. Find friction coefficients for various material combinations, including static and kinetic friction values. Useful for engineering, physics, and mechanical design applications. Calculate fuel consumption in miles per gallon - mpg - calculator and consumption charts. Impact forces acting on falling objects hitting the ground, cars crashing and similar cases. Forces acting a very short time are called impulse forces. Energy possessed by an object's motion is kinetic energy. Compare the total cost of owning a new car versus an old car, including purchase price, maintenance, fuel efficiency, insurance, and depreciation. Use our detailed breakdown to make an informed financial decision before buying the next vehicle. Cylinder volume and compression ratios in piston engines. Calculate piston engine displacement. Speed (mph) and time (hours) and distance traveled (miles) chart. Speed (km/h) vs. time (hours) and distance traveled (km). Traffic flow and density as used in highway design. Related Resources: mechanics machines Coefficients of Rolling Friction for Wheels with Steel and Pneumatic Tires McKibben and Davidson (Agri. Eng., 1939) give the data in Table calculator below on the rolling resistance of various types of wheels for typical road and field conditions. Note that the coefficient fr is the ratio of resistance force to load. Moyer found the following average values of fr for pneumatic rubber tires properly inflated and loaded: hard road, 0.008; dry, firm, and well packed gravel, 0.012; wet loose gravel, 0.06. Rolling Resistance Source: Marks Standard Handbook for Mechanical Engineers Related This paper focuses on developing a relationship between the rolling resistance coefficient of an off-road truck tire running over different terrains and at various operating conditions. The various operating conditions include tire speed, vertical load, and inflation pressure. The different terrains include dry and moist sand, flooded surface, snow, dense sand, clayey soil, and sandy loam with moisture. The off-road truck tire size 315/80R22.5 is modeled and validated using the finite element analysis technique, while the terrains are modeled and calibrated using smoothed-particle hydrodynamics technique. Artificial neural network and genetic algorithm (GA) are then used to develop a relationship between the rolling resistance coefficient and the terrains and operating conditions. The results of both algorithms are compared to the simulation results in terms of R-square goodness of fit and the mean squared error. Finally, a numerical equation is presented that determines the rolling resistance coefficient as a function of the terrains parameters and the operating conditions. It was found that both techniques provide a suitable solution, however, the GA provides an explicit equation. Use our pre-submission checklist Avoid common mistakes on your manuscript. The optimization process has been applied to the field of tire mechanics since 1973[1]. The genetic algorithm (GA) was first introduced by John Holland[2] in 1970s to duplicate the nature actions. Holland was anxious about algorithms that employ binary digits. The chromosome used in the GA is an artificial one that has a number of genes and each one is represented using binary numbers of 0 or 1. It was noticed that nature has the capability to learn and adapt arbitrary, in other words the nature finds and selects good chromosomes. The GA algorithm is designed to duplicate the nature ability, it employees two mechanisms to solve problems, the encoding and evaluation. The GA relies on reproduction, crossover, selection and mutation which are duplicated by mathematical models. A measure of fitness is implemented for each individual chromosome to accomplish reproduction. During the reproduction phase, the crossover operator is implemented to exchange elements of two chromosomes. Later, the gene value in some of the chromosomes that are randomly chosen are adjusted using the mutation operator.In 2000, Licheng[3] developed a novel GA algorithm based on immunity, Licheng utilized the local information in order to avoid repetitive work, it was concluded that the novel algorithm is effective and improves the searching ability. Later in 2004, Abe[4] used the GA to optimize the construction of the tire reinforcement. Abe indicated that the if one characteristic is enhanced the quality of another character is affected by sacrificing it. Abe concluded that using the GA the cornering stiffness of the tire increases more than 15% in comparison with an indoor drum test performed in control structured environment.In 2008, Matuvsko[5] implemented the artificial neural network (ANN) to estimate the tireroad frictional forces. Matuvsko added the ANN to the estimator in an attempt to model friction effect, it was concluded that the estimation of the forces was very fast and the system was able to compensate for the change of the model parameters.In 2012, Cho et al.[6] modified the stiffness and shape of the sidewalls of a run-flat tire in an attempt to investigate the durability and riding comfort. Cho developed a genetic algorithm to replace the CPU time consuming simulations. Cho emphasized on the idea of the run-flat tires and the importance of protecting the sidewall when the pressure level becomes low and verified his proposed method successfully seeks Pareto solutions. In the same year, Cirovic[7] used the Recurrent Neural Network to design a braking torque controller. The primary objective of his research was to model the dynamic change in the braking torque in order to develop a dynamic model able to capture the braking cycle. Cirovic concluded that the braking torque could be dynamically modeled to predict the braking torque under different conditions including applied pressure, the speed and the interface temperature during a braking cycle.In 2017, Zhu et al.[8] designed a smart tire to estimate the tire forces and the influence of specific factors. The Neural Network algorithm was applied to determine the tire forces and an FTire model was used. It was concluded that the proposed method is efficient and simple.ANN is an interconnected systems of simple processing units. The processing units are referred to as neurons and they work in parallel to achieve a required result. In most applications the ANN is regarded to as black box that has an input and output. Figure1 shows the structure of an ANN which is based on the original model developed in 1943 by Mcculloch[9]. The model is an attempt to model the signal processing characteristics of the biological nerve cell in a mathematical approach. It is noted that \(w_i\) is the weight of each input channel, \(p_i\), y is the output, and n is the number of neurons. The weight of each channel increases or decreases the incoming signal.Fig. 1A model presenting the artificial neuron[9]The purpose of this paper is to utilize both the ANN and GA algorithms to numerically model a relationship between the rolling resistance coefficient and tireterrain operating conditions. The tireterrain operating conditions include tire speed, vertical load, inflation pressure, in addition to the terrain cohesion, angle of shear resistance and depth. The results of this paper are beneficial to future tireterrain interaction research and implementation of tire model in full vehicle model. In addition, the developed equation will further allow to replace the rigid ring tire models by better non-linear relationships.The tire modeling and validation using Finite Element Analysis (FEA) technique is explained. Then, the smoothed-particle hydrodynamics (SPH) technique to model soft terrains is discussed. Finally, the tireterrain interaction algorithm and the rolling resistance model set-up is explained.2.1 FEA tire modelThe FEA truck tire used in this research is modeled in PamCrash a multiphysics simulation software package and shown in Fig.2. Furthermore, the FEA truck tire model developed in Pam-Crash is validated in static domain by performing the vertical stiffness and footprint tests and in dynamic domain by performing the drum-cleat test.Fig. 2315/80R22.5 FEA truck tire dimensions[10]The tires vertical stiffness , \(k_{tot}\), is calculated during the vertical stiffness test. The truck tire is strained in all translational and rotational directions apart from the vertical translational direction. The vertical rest starts by applying a low rate ramp vertical load to the center of the tire resulting in deformation. The relationship between the applied vertical load and the tires center deflection is computed and the tire vertical stiffness, \(k_{tot}\), is calculated using Eq.1. The tire spring rate is the validated against provided manufacturer data.$$k_{tot}= \frac{\varDelta load}{\varDelta deflection}$$The contact patch of the tire with the ground is computed using the static footprint test as shown in Fig.3. The same test procedure used in the vertical stiffness test is adapted in this test as well. Nonetheless, the contact area of the tire with the ground is recorded and compared to measured data at a rated inflation pressure of 758kPa (110psi). The measured contact area provided from measured data is about 450 \({\rm cm}^2\)[11], while the calculated contact area is 498 \({\rm cm}^2\), with a difference less than 10%.Fig. 3Normal nodal velocity acting on the tireroad contact area at 27kN vertical load and 758kPa inflation pressureThe radial tire stiffness causes the tire radius to vary depending on the material characteristics and the tire inflation pressure. The drum-cleat test is used to compute the first mode frequency. The truck tire is excited over rigid circular drum with a cleat on top[12]. The vertical forces are computed and then the Fast Fourier Transformation (FFT) algorithm is deployed to convert the fores into frequency domain. The computed first model of vibration is then validated against measured data[12].The predicted FEA truck tire vertical stiffness, \(k_{tot}\) in kN/m and the first mode of vibrations in Hz at an applied vertical load of 27kN (6000lbs) and several inflation pressures are shown in Table1.Table 1 FEA truck tire predicted parameters at 27kN and different inflation pressures[11]2.2 SPH terrain modelThe particle is described as a sphere centred on the centre of mass, each particle has a radius, a mass, and a velocity. The discretized conservation equations governs the change of particle characteristics. Each particle also has three Degrees of Freedom (DOF), namely the centre of mass of the particle, the volume of the particle, and the domain of influence[13].The smoothing length, h, is used to define the distance between two consecutive particles, while the minimum smoothing length is defined as \(H_{min}\) and the maximum smoothing length is defined as \(H_{max}\). The Anti-Crossing force (ETA) relationship is shown in Eq.2, where \(\epsilon\) is the relative strength which is generally less than 0.5, \(\rho\) is the density of the particles and \(W_{ij}\) is the weighing function[11].$$ETA= u_i + \epsilon \sum _j m_j \frac{ u_j - u_i}{\frac{1}{2} (\rho _i + \rho _j)} W_{ij}$$In this research, several terrains are modeled and calibrated to predict the tireterrain interaction. The terrain modeled include soils such as dry and moist sand, clayey soil, dense sand and snow which are modeled using the isotropicelasticplastichydrodynamic relationship[10, 11, 14], while other terrains includes the water to model flooded surface[15,16,17]. Furthermore, the moisture effect of the soils is also taken into account by modeling moist sand and sandy loam with different moisture content[18].Furthermore, Murnaghan equation of state for solid elements is used to model water and flooded surfaces. The equation of state requires the input of the density \(\rho\), the bulk coefficient B, and the exponent \(\gamma\), in addition to viscosity coefficients. The Murnaghan equation of state relationship is described in Eq.3, the exponent \(\gamma\) is the density ratio exponent which is generally equal to 7, and the ratio \(\rho /\rho _0\) is the current to the initial mass densities. Equation4 is used to determine the coefficient B which is related to the speed of sound.$$p= p_0 + B \left({\left(\frac{\rho }{\rho _0} \right) }^\gamma -1 \right)$$$$B\ge 100 \rho _0 \frac{{V_{max}}^2}{\gamma }$$The isotropicelasticplastichydrodynamic material was used to model the soil behavior. Equation5 shows the Equation of State (EOS) for isotropicelasticplastichydrodynamic material which governs the pressurevolume relationship at low pressure. Where, \(c_0\) to \(c_6\) are EOS coefficients, \(\mu\) is the ratio of current over initial mass density and is written as \(\mu =\rho /\rho _0-1\), and \(E_i\)is the internal energy[13]. The material properties used in this research are obtained from published terramechanics research, these properties are implemented in the soil definition to calibrate the soil characteristics.$$p= c_0 + c_1 \mu + c_2 \mu ^2 + c_3 \mu ^3 + (c_4 + c_5 \mu + c_6 \mu ^2) E_i$$The pressuresinkage and the shearstrength tests are performed to calibrate the modeled soil. The results of the modeled soil are calibrated against published terramechanics data.2.2.1 Pressuresinkage testA constant pressure is applied to a circular plate of 150mm radius, the plate is located on top of a \(800\times 800\times 800\)mm box filled with soil domain shown in Fig.4. The soil domain is exposed to a range of pressures between 0 to 200kPa and the sinkage of the plate is recorded for each pressure at steady state. The results obtained from the pressuresinkage test are calibrated against Bekkers relationship presented in Eq.6. Where p is the applied pressure in kPa, b is radius of the circular plate in mm, z is the plate sinkage in mm, and n, \(k_c\), and \(k_\theta\) are soil parameters obtained from published terramechanics research.$$p=\left(\frac{k_c}{b}+k_\theta \right) z^n.$$Fig. 4Pressuresinkage test[10]2.2.2 Shearstrength testA soil domain is filled inside a rectangular box, as displayed in Fig.5. The box consists of three components the top pressure plate where pressure is applied, the upper box which is the sliding component and the fixed bottom box which is fixed in all directions. A constant pressure is applied to the centre of the top pressure plate, then a 10m/s displacement is applied to the upper box and the top plate. The shear force is recorded and the shearstress relationship is obtained. The shear characteristics of the soil obtained form the shearstress relationship is calibrated against Janosi and Hanamoto exponential function shown in Eqs.7 and8.$$\tau= \tau _{max }\left(1-e^{(-j/k)} \right) = (c+p \tan {\phi }) \left(1-e^{(-j/k)} \right)$$MohrCoulomb failure criterion shown in Eq.8 is used to determine the maximum shear \(\tau _{max}\), where c is the soil cohesion in kPa and \(\phi\) is the angle of shear resistance in degrees obtained from published terramechanics data.$$\tau _{max} =c+p \tan {\phi }$$Fig. 5The calibration procedure requires the repetition of both tests using exact material properties to obtain realistic soil characteristics. In the course of the calibration and validation, the material properties are repeatedly adjusted, and the pressuresinkage and shearstrength calibration tests are repeated until the simulation results and the terramechanics data are in best agreement.2.3 Rolling resistance analysisThe rolling resistance analysis is performed under different operating conditions and several terrains to determine the effect on the tireterrain interaction as shown in Fig.6. The results of the simulations are then used in a ANNGA algorithm to develop a numerical relationship between the rolling resistance coefficient and the terrain parameters and operating conditions.Fig. 6Schematic of a tire during rolling resistance test over dry sandThe rolling resistance simulation is common for all operating conditions. A pressure is applied to the tire components to obtain a specified inflation pressure, then a rapid constant vertical load is applied to the centre of the tire. The tire is then left on the terrain to stabilize. Later, a constant linear longitudinal velocity is applied to the centre of the tire. The tire then rolls over the terrain for about 1.5s to obtain steady forces at the contact patch. The rolling resistance coefficient, \(f_r\), is then calculated as shown in Eq.9, where \(F_x\) and \(F_z\) are the average forces at steady state, in the longitudinal and vertical directions, respectively.$$f_r = \frac{F_x}{F_z}$$The performed simulation to predict rolling resistance is repeated at several terrains including flooded surface, snow, dry and moist sand, clayey soil, dense sand, and snow. The obtained results are validated and published[10, 11, 17]. The simulations are repeated at different operating conditions such as; inflation pressures (379kPa, 586kPa and 758kPa); applied vertical loads (13kN, 27kN, and 40kN); and tire speed ranging between 10 and 100km/h.Fig. 7Rut depth left in different terrains for tire running at 10km/h and 0.5sFigure7 shows the rut depth in the several terrains for the tire running at 10km/h speed, 27kN vertical load and 586kPa inflation pressure. It is observed that rut depth left in soil depends on the terrain type and moisture content, for instance the rut left in dry sand around 225mm which is less than that left in clayey soil of around 157.5mm. Also, in comparison between dry sand which has a rut depth of 225mm and 10% moist sand which has a rut depth of 344mm, the moist sand has a greater rut depth.In the case of snow, the rut depth recorded was around 300mm this is due to the fact that the tire in snow causes a splash phenomenon which causes some particles of the snow to splash away of the box. It is also noticed that in cases such as clayey soil and snow due to the high sinkage of the tire in the terrain the front zone of the terrain also display before the tire is in contact, this is also referred to as bulldozing effect.The ANN and the GA methods are used to numerically relate the rolling resistance coefficient of the truck tire running over the previously mentioned operating conditions. The purpose of this research is to evaluate the results obtained from both methods, and to develop a relationship between the rolling resistance coefficient,\(f_r\), the terrain parameters and the operating conditions in order to determine the rolling resistance coefficient without the need to perform simulations.The terrains used include dry sand, dense sand, clayey soil, water, snow, sandy loam with moisture content and moist sand. The terrains are presented using the cohesion, C, in kPa angle of shear resistance, \(\phi\), in degrees and depth, D, in mm while the tire operating conditions are presented using the inflation pressure, P, in kPa vertical load, L, in kN and speed, V in m/s. Thus, each output rolling resistance coefficient, \(f_r\), value is associated with six inputs, the input vector X is defined as shown in Eq.10.$$X^T = \begin{bmatrix} L&P&V&C&\phi&D \end{bmatrix}$$3.1 Artificial neural networkTwo-layer of fully-interconnected feed-forward ANN are used as shown in Fig.8. A fully-interconnected feed-forward ANN indicates that each input is connected to each neuron and the data is processed from left to right which is the forward direction. The parameter \(b_i\) is the bais which is shown in Fig.1.Fig. 8Diagram of a two-layer fully-interconnected feed-forward ANNIn the case of a single neuron, the relationship between the input and the output is shown in Eq.11, where f is the activation function performing the summation of weighted inputs. Equation11 can be expressed in matrix form as shown in Eq.12. Where W and P are expressed as shown in Eqs.13 and14, respectively.$$y= f \left(w_1 p_1 + w_2 p_2 + w_3 p_3 \ldots w_n p_n +b \right)$$$$y= f \left(W P + b \right)$$$$W= \left[w_1 \quad w_2 \quad w_3 \quad \ldots \quad w_n \right]$$$$P^T= \left[p_1 \quad p_2 \quad p_3 \quad \ldots \quad p_n \right] ^T$$In a general form for an ANN with k number of layers the desired output is written as shown in Eq.15.$$Y^k = f^k \left(w^k f^{k-1} \left(W^{k-1} f^{k-2} \left(W^{k-2} f^{k-3} (\ldots)+ B^{k-3}\right) + B^{k-2}\right) + k^{k-1}\right) + B^{k} \right)$$where the matrices \(Y^k\), \(B^k\), and \(W^k\) are defined as shown in Eqs.16,17, and18, respectively.$$Y^k= \left[y _1^k\quad y _2^k\quad y _3^k\quad \ldots \quad y _z^k \right] ^T$$$$B^k= \left[b _1^k\quad b _2^k\quad b _3^k\quad \ldots \quad b _z^k \right] ^T$$$$W^k= \begin{bmatrix} w _{1,1}^k &{} w _{1,2}^k &{} w _{1,3}^k &{}\ldots &{} w _{1,n}^k \\ w _{2,1}^k &{} w _{2,2}^k &{} w _{2,3}^k &{}\ldots &{} w _{2,n}^k \\ w _{3,1}^k &{} w _{3,2}^k &{} w _{3,3}^k &{}\ldots &{}w _{3,n}^k\\ \vdots &{} \vdots &{} \vdots &{} \ddots &{} \vdots \\ w _{m,1}^k &{} w _{m,2}^k &{} w _{m,3}^k &{}\ldots &{} w _{m,n}^k \end{bmatrix}$$The ANN is implemented in MATLAB using the Neural Net Fitting tool, the number of layers was chosen to be two, with tan-sigmoid neurons in the hidden layer and linear neurons in the output layer which is a commonly used function approximation. The number of neuron in each layer was set to 10. Generally, the equation that describes the input-output mapping for an ANN with tan-sigmoid neurons derived from Eq.12 in the hidden layer and linear neurons in the output is shown in Eq.19. Where Y is the output vector which is in this case the RRC [\(1\times 1\)], X is the input vector which is in this case [\(6\times 1\)], LW is the first layer weight matrix with size [\(10\times 6\)], IW is the second layer weight matrix with size [\(1\times 10\)], and \(b_1\) and \(b_2\) are the first and second layer bias vectors, respectively. It should be noted that since 10 neurons were deployed during the definition process the vector size \(b_1\) has the size [\(10 \times 1\)] and the vector \(b_2\) is [\(1 \times 1\)].$$Y= LW \left({\text {tansig}} \left(IW X + b_1 \right) \right) +b_2$$The tansig function is defined below:$$ {\text {tansig}} (IW X + b_1) = \frac{2}{1+e^{-2\left(IW X + b_1 \right) }} -1$$After collecting all the rolling resistance coefficient data from simulations an input of [\(6 \times 147\)] and the vector of [\(1 \times 147\)] were imported into MATLAB Neural Net Fitting toolbox. Consequently, an ANN training was performed and the bias vector and weight matrix were determined. Details about the ANN fitting and equation are given in Appendix. Figure9 shows the variation of the observed rolling resistance coefficient as a function of the predicted.Fig. 9Observed as a function of predicted rolling resistance coefficient for an R-square fitnessThe mean square error (MSE) was computed to be \(5e^{-5}\) and the R-square goodness of fit is 0.9974. It is clearly observed that the observed and predicted rolling resistance coefficients falls within the same values and are very close to each others over the whole range. The observed and predicted rolling resistance coefficients are plotted in a range of the line with equation \(y=x\) which shows the perfect fitting between both values. It is concluded that the ANN equation successfully predicts the rolling resistance coefficient for the range of the 6 inputs (inflation pressure, vertical load, terrain cohesion, terrain shear resistance, terrain depth, and tire speed).3.2 Generic algorithmThe GA is utilized in this research to find the relationship between various parameters. A basic GA has 10 simple steps, first a chromosome of a fixed length is used to represent the problem variable domain, then the size of the chromosome population, N is chosen. Second, the fitness function is defined to measure the fitness of each individual chromosome. Third, the initial population of chromosomes of size N is randomly generated. Fourth, the fitness of each individual chromosome is computed. Fifth, a pair of chromosomes is then selected for mating from the current population, the selection process is based on a fitness probability. Sixth, a pair of offspring chromosomes is created by implementing the crossover and mutation. Seventh, created offspring chromosomes are placed in the new population and fifth step is repeated until the new chromosome population size is equal to the initial population size. Then, the initial chromosome population is replaced with the new offspring one. Finally, all steps from step four are repeated until the termination criterion is fulfilled.The fitness functions are well known in the optimization process. The objective function of the optimization problem is generally defined by Eq.21, where \(\phi _i^*t\) is the penalty function at the ith string in the tth generation, \(X _i^*t\) is the vector of the design variable and N as mentioned before is the population size.$$\phi _i^t = f\left({X _i}^t \right) + \sum _{j=1}^ {m}\beta _j max \left(0,g _j \left({X _i}^t \right) \right) \quad (i=1,\ldots N)$$Implementing the same technique used for ANN and using the same inputs and outputs obtained from simulations an equation relating the input parameters to the rolling resistance coefficient, \(f_r\), as a function of the input dynamics techniques, where \(k_1\), \(k_2\), \(k_3\), \(k_4\) are constant equal to 0.0876, 0.167, 1.08e\(-\)5 and 0.001, respectively.$$f_r = k_1 V*D + k_2 D^2 + k_3 L*P*D^2 + k_4 \frac{L}{\phi -12.4} + \frac{\phi D}{L*C^2}$$Figure10 presents the observed as a function of the predicted rolling resistance coefficient. The R-square goodness measures how close the data are to the fitted regression line as shown in Eq.23.Fig. 10Observed as a function of the predicted rolling resistance coefficient for an R-square fitnessFor example a value of 0 imply that the model demonstrate none of the variability of the response data around its means, while a value of 1 imply that the model demonstrate total variability of the response data around the mean. Thus generally the higher the \(R^2\) goodness the better. The calculated R-square of Eq.22 is 0.94.$$R^2 = \frac{\text {Explained variation}}{\text {Total variation}}$$The Mean Absolute Error (MAE) is used to determine the mean of the absolute value of residual errors and is defined in Eq.24, n is the number of data points, y is the hydroplaning speed input and f(x) is the hydroplaning speed predicted by the equation.$$MAE = \frac{1}{n} \sum _{i=1}^{n} \mid y-f(x) \mid$$The MAE calculated for Eq.22 using Eq.24 is 0.021. Furthermore, the Mean Squared Error (MSE) is used to minimize the mean of the squared residual errors and is defined in Eq.25.$$MSE = \frac{1}{n} \sum _{i=1}^{n} \left(y-f(x) \right) ^2$$The calculated MSE is 0.0012 based on Eq.25. It is noticed that the observed and predicted rolling resistance coefficients falls within a similar range and clearly lies in the range of the equation line \(y=x\). However, for a low rolling resistance coefficient value less that 0.1 the results are slightly scattered.3.3 Testing of algorithms performanceIn comparison between Figs.9 and10 it is observed that the fitting of the observed versus the predicted rolling resistance coefficient of the neural network is better than that of the genetic algorithm. However, on the other side the complexity of neural network equation mentioned in Eq.34 is way higher than that of the genetic algorithm equation mentioned in Eq.22. Thus a compromise between the complexity of the equation and the fitness should be made depending on the application purpose.To further estimate the performance of the ANN and GA algorithms, Fig.11 is presented. Figure11 shows the variation of the rolling resistance coefficient as a function of the tire speed for a tire running over 200mm snow depth, 27kN vertical load and 586kPa inflation pressure for simulations, ANN and GA algorithms. All three lines shows the same pattern as the tire speed increases the rolling resistance coefficient increases. However, the ANN algorithm better predicts the simulation results in comparison to the GA predictions. Overall, the GA predictions are at a maximum error of 0.06.Fig. 11rolling resistance coefficient as a function of tire speed for simulations, ANN and GA algorithmsFurthermore, the variation of the rolling resistance coefficient at 27kN vertical load, 586kPa inflation pressure, 10km/h tire speed and a soil depth of 585mm as a function of the soil cohesion is represented in Fig.12. Generally, the rolling resistance coefficient for all three curves exhibits the same trend. However, for a low soil cohesion the ANN predictions are closer to those of the simulations in comparison to that of the GA predictions. At higher cohesion such as 15kPa both ANN and GA are within a maximum error of 0.025.Fig. 12Rolling resistance coefficient as a function of soil cohesion for simulations, ANN and GA algorithmsThe variation of the rolling resistance coefficient at 27kN vertical load, 586kPa inflation pressure, 10km/h tire speed and a soil depth of 585mm as a function of soil angle of shear resistance angle is represented in Fig.13. Generally, all three lines has the same trend, as the angle of shear resistance increase the rolling resistance coefficient decrease. Moreover, at low angle of shear resistance both ANN and GA have a maximum error of 0.03, at high angle of shear resistance the lines are coinciding.Fig. 13Rolling resistance coefficient as a function of soil angle of shear resistance for simulations, ANN and GA algorithmsA Finite Element Analysis off-road truck tire size 315/80R22.5 was modeled and validated. The rolling resistance coefficient of the truck tire running over various terrains at different operating conditions was computed. The terrains were modeled using smoothed-particle hydrodynamics technique and calibrated and validated against published terramechanics data. The various terrains included dry and moist sand, clayey soil, flooded surface, snow, dense sand and sandy loam with moisture content. The different operating conditions included inflation pressures, vertical loads, and tire speeds.The rolling resistance coefficient results obtained from several FEASPH simulations were used to built a numerical relation between the terrain and operating conditions. The data collected was used as an input/output to the algorithms with the terrain parameters (cohesion, angle of shear resistance, depth) and the operating conditions (tire inflation pressure, applied vertical load and tire speed) as an input and the rolling resistance coefficient as an output.Two relationships were built using the artificial neural network (ANN) and the genetic algorithm (GA) methods. The mean square error of the ANN and GA algorithms was computed to be \(5e^{-5}\) and 0.0012, respectively. While the R-square goodness of fit was 0.9974 for the ANN and 0.94 for the GA. It is concluded that the ANN has a better fitting in regards to the observed versus predicted rolling resistance coefficient. However, the GA has a better numerical equation in terms of complexity.The main concluding remarks are presented as follows: A hybrid FEA-SPH model to simulate the tireterrain interaction was developed and validated. The rolling resistance coefficient of the truck tire running over various terrains at various operation condition were collected. Two relationships using the ANN and GA were developed to predict the rolling resistance coefficient as a function of operation conditions. The ANN developed relationship has a better fitting in regards to the observed versus predicted rolling resistance coefficient. The GA developed relationship is a better relationship in terms of complexity. The develop equation is based on the set of simulations performed and is limited to the truck tire used. It would be beneficial for further studies to test and validate the equation on different tires and various operating conditions. Schmit LA, Farshi B (1973) Optimum laminate design for strength and stiffness. Int J Numer Methods Eng 7(4):519536Article Google Scholar Holland JH (1992) Genetic algorithms. Sci Am 267(1):6673Article Google Scholar Jiao L, Wang L (2000) A novel genetic algorithm based on immunity. 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The bias and weight of the ANN were determined as follows:$$b_1^{T} =[\begin{array}{cccccccccc} 3.0&-1.5&2.1&-0.36&0.73&-0.11&0.48&-0.47&0.65&1.5 \end{array}]$$ (26) $$IW= \begin{bmatrix} 0.058 &{} -0.023 &{} -0.011&{} -1.5&{} 3.9&{} -1.2 \\ 0.81&{} 0.069&{} 2.1&{} -0.68&{} -1.0&{} 1.9 \\ -0.68&{} -0.043&{} -0.72&{} 1.7&{} -0.32&{} -0.2\\ -0.033&{} -0.085&{} -1.1&{} 1.6&{} -0.048&{} -2.0\\ 0.19&{} -0.58&{} -1.1&{} -0.74&{} -1.1&{} -1.9\\ 0.11&{} -0.029&{} 0.69&{} 1.5 &{} 2.2&{} 0.65\\ -0.079 &{} 2.8&{} -1.3&{} -1.8&{} -1.4&{} -3.5&{} -1.5\\ -0.61&{} 0.018&{} 1.0&{} -0.032&{} -0.011&{} -0.54\\ 0.84&{} -0.023&{} 0.48\\ -1.0&{} -0.02&{} -1.2&{} 3.2&{} 2.2&{} 1.1&{} 0.67&{} 0.43&{} 0.48\\ -1.0&{} -0.02&{} -1.2 \end{bmatrix}$$ (28) $$LW= [\begin{array}{cccccccccc} 2.1&1.2&0.55&-0.31&-0.19&2.6&-3.8&-1.4&-0.94&1.0 \end{array}]$$ (29) $$b_2 = 0.29$$ (30) Let M be defined as show in Eq.30, in which the tansig is a ANN transfer function that calculates the layers output from its net input as defined as shown in Eq.31.$$\text {tansig} (IW X + b_1) = \frac{2}{1+e^{-2\left(IW X + b_1 \right) }} -1$$ (31) Substituting IW, \(b_1\) and the input vector X Eq.30 is rearranged and presented in32.$$M = \begin{bmatrix} 0.058 L - 1.2 D - 1.5 C - 0.023 P - 0.011 V + 3.9 \phi + 3 \\ 1.9 D - 0.68 C + 0.81 L + 0.07 P + 2.1 V - \phi - 1.5 \\ 1.7 C - 2 D - 0.68 L - 0.043 P - 0.72 V - 0.32\phi + 2.1\\ 1.6 C - 2 D - 0.033 L - 0.085 P - 1.1 V - 0.048 \phi - 0.36\\ 0.19 L - 1.9 D - 0.74 C - 0.58 P - 1.1 V - 1.1 \phi + 0.73\\ 1.5 C + 0.65 D + 0.11 L - 0.029P + 0.69V + 2.2\phi - 0.11\\ 2.8e{-}3 P - 1.5 C - 0.079 L - 1.4 C - 1.8 V - 3.5 \phi + 0.48\\ 0.018 P - 0.54 D - 0.61 C - 0.032 C + 1.0 V - 0.011\phi - 0.47\\ 0.67 C + 0.48 D + 0.84 L - 0.023 P + 1.8 V + 0.43 \phi + 0.65\\ 3.2 C + 1.4 D - 1.1 L - 0.02 P - 1.2 V + 2.2 \phi + 1.1 \end{bmatrix}$$ (32) Substituting and simplifying Eq.32 in Eq.31, Eq.33 is obtained.$$\text {tansig} (M) = \begin{bmatrix} \frac{2}{1+\exp \left(3 C + 2.3 D - 2.12 L + 0.047 P + 0.021 V - 7.9 \phi - 6\right) }-1\\ \frac{2}{1+\exp \left(1.4 C - 3.7 D - 1.6 L - 0.14 P - 4.1 V + 2.0 \phi + 2.9 \right) }-1\\ \frac{2}{1+\exp \left(0.4 D - 3.4 C + 1.4 L + 0.086 P + 1.4 V + 0.65 \phi - 4.1\right) }-1\\ \frac{2}{1+\exp \left(4.1 D - 3.3 C + 0.066 L + 0.17 P + 2.1 V + 0.095 \phi + 0.72\right) }-1\\ \frac{2}{1+\exp \left(1.5 C + 3.7 D - 0.39 L + 2.2 P + 2.2 V + 2.3 \phi - 1.5\right) }-1\\ \frac{2}{1+\exp \left(0.059 P - 1.3 D - 0.23 L - 3.1 C - 1.4 V - 4.4 \phi + 0.21\right) }-1\\ \frac{2}{1+\exp \left(2.8 C + 3.1 D + 0.16 L - 5.7e{-}3 P + 3.6 V + 7 \phi - 0.95\right) }-1\\ \frac{2}{1+\exp \left(0.063 C + 1.1 D + 1.2 L - 0.036 P - 2.1 V + 0.023 \phi + 0.93\right) }-1\\ \frac{2}{1+\exp \left(0.047 P - 0.96 D - 1.7 L - 1.3 C + 3.7 V - 0.86 \phi - 1.3\right) }-1\\ \frac{2}{1+\exp \left(2 L - 2.9 D - 6.3 C + 0.041 P + 2.4 V - 4.4 \phi - 3\right) }-1 \end{bmatrix}$$ (33) The Y output in Eq.19 which is in this case the rolling resistance coefficient, \(f_r\), becomes as shown in Eq.34 after solving using Eq.33.$$\begin{aligned} f_r & = \frac{2.1}{\exp \left(2 L - 2.9 D - 6.3 C + 0.041 P + 2.4 V - 4.4 \phi - 3\right) + 1} \nonumber \\&\quad - \frac{0.63}{\exp \left(4.1 D - 3.3 C + 0.066 L + 0.17 P + 2.1 V + 0.095 \phi + 0.72\right) + 1} \nonumber \\&\quad -\frac{ 2.9}{\exp \left(0.063 C + 1.1 D + 1.2 L - 0.036 P - 2.1 V + 0.023 \phi + 0.93\right) + 1} \nonumber \\&\quad+ \frac{2.4}{\exp (1.4 C - 3.7 D - 1.6 L - 0.14 P - 4.1 V + 2 \phi + 2.9) + 1} \nonumber \\&\quad+ \frac{ 0.39}{\exp (1.5 C + 3.7 D - 0.39 L + 2.2 P + 2.2 V + 2.3 \phi - 1.5) + 1} \nonumber \\&\quad+ \frac{1.1}{\exp (0.4 D - 3.4 C + 1.4 L + 0.086 P + 1.4 V + 0.65 \phi - 4.1) + 1} \nonumber \\&\quad+ \frac{ 5.2}{\exp \left(0.059 P - 1.3 D - 0.23 L - 3.1 C - 1.4 V - 4.4 \phi + 0.21\right) + 1} \nonumber \\&\quad+ \frac{ 4.1}{\exp (3 C + 2.3 D - 2.12 L + 0.047 P + 0.021 V - 7.9 \phi - 6) + 1} \nonumber \\&\quad+ \frac{ 7.7}{\exp \left(2.8 C + 3.1 D + 0.16 L - 5.7e{-}3 P + 3.6 V + 7 \phi - 0.95\right) + 1} \nonumber \\&\quad- \frac{1.9}{\exp \left(0.047 P - 0.96 D - 1.7 L - 1.3 C + 3.7 V - 0.86 \phi - 1.3\right) + 1} \nonumber \\&\quad - 10.7 \end{aligned}$$ (34) However, Eq.34 can not be simplified anymore and this is the final form that is adopted to compute the rolling resistance coefficient at different operating conditions.

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